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Preface

Welcome to *Prealgebra*, an OpenStax resource. This textbook was written to increase student access to high-quality learning materials, maintaining highest standards of academic rigor at little to no cost.

About OpenStax

OpenStax is a nonprofit based at Rice University, and it's our mission to improve student access to education. Our first openly licensed college textbook was published in 2012, and our library has since scaled to over 20 books for college and AP® Courses used by hundreds of thousands of students. Our adaptive learning technology, designed to improve learning outcomes through personalized educational paths, is being piloted in college courses throughout the country. Through our partnerships with philanthropic foundations and our alliance with other educational resource organizations, OpenStax is breaking down the most common barriers to learning and empowering students and instructors to succeed.

About OpenStax Resources

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Format

You can access this textbook for free in web view or PDF through openstax.org.

About *Prealgebra*

Prealgebra is designed to meet scope and sequence requirements for a one-semester prealgebra course. The text introduces the fundamental concepts of algebra while addressing the needs of students with diverse backgrounds and learning styles. Each topic builds upon previously developed material to demonstrate the cohesiveness and structure of mathematics.

Students who are taking *Basic Mathematics* and *Prealgebra* classes in college present a unique set of challenges. Many students in these classes have been unsuccessful in their prior math classes. They may think they know some math, but their core knowledge is full of holes. Furthermore, these students need to learn much more than the course content. They need to learn study skills, time management, and how to deal with math anxiety. Some students lack basic reading and arithmetic skills. The organization of *Prealgebra* makes it easy to adapt the book to suit a variety of course syllabi.

Coverage and Scope

Prealgebra follows a nontraditional approach in its presentation of content. The beginning, in particular, is presented as a sequence of small steps so that students gain confidence in their ability to succeed in the course. The order of topics was carefully planned to emphasize the logical progression throughout the course and to facilitate a thorough understanding of each concept. As new ideas are presented, they are explicitly related to previous topics.

Chapter 1: Whole Numbers

Each of the four basic operations with whole numbers—addition, subtraction, multiplication, and division—is modeled and explained. As each operation is covered, discussions of algebraic notation and operation signs, translation of algebraic expressions into word phrases, and the use the operation in applications are included.

Chapter 2: The Language of Algebra

Mathematical vocabulary as it applies to the whole numbers is presented. The use of variables, which distinguishes algebra from arithmetic, is introduced early in the chapter, and the development of and practice with arithmetic concepts use variables as well as numeric expressions. In addition, the difference between expressions and equations is discussed, word problems are introduced, and the process for solving one-step equations is modeled.

Chapter 3: Integers

While introducing the basic operations with negative numbers, students continue to practice simplifying, evaluating, and translating algebraic expressions. The Division Property of Equality is introduced and used to solve one-step equations.

Chapter 4: Fractions

Fraction circles and bars are used to help make fractions real and to develop operations on them. Students continue simplifying and evaluating algebraic expressions with fractions, and learn to use the Multiplication Property of Equality to solve equations involving fractions.

Chapter 5: Decimals

Basic operations with decimals are presented, as well as methods for converting fractions to decimals and vice versa. Averages and probability, unit rates and unit prices, and square roots are included to provide opportunities to use and round decimals.

Chapter 6: Percents

Conversions among percents, fractions, and decimals are explored. Applications of percent include calculating sales tax, commission, and simple interest. Proportions and solving percent equations as proportions are addressed as well.

Chapter 7: The Properties of Real Numbers

The properties of real numbers are introduced and applied as a culmination of the work done thus far, and to prepare students for the upcoming chapters on equations, polynomials, and graphing.

Chapter 8: Solving Linear Equations

A gradual build-up to solving multi-step equations is presented. Problems involve solving equations with constants on both sides, variables on both sides, variables and constants on both sides, and fraction and decimal coefficients.

Chapter 9: Math Models and Geometry

The chapter begins with opportunities to solve “traditional” number, coin, and mixture problems. Geometry sections cover the properties of triangles, rectangles, trapezoids, circles, irregular figures, the Pythagorean Theorem, and volumes and surface areas of solids. Distance-rate-time problems and formulas are included as well.

Chapter 10: Polynomials

Adding and subtracting polynomials is presented as an extension of prior work on combining like terms. Integer exponents are defined and then applied to scientific notation. The chapter concludes with a brief introduction to factoring polynomials.

Chapter 11: Graphs

This chapter is placed last so that all of the algebra with one variable is completed before working with linear equations in two variables. Examples progress from plotting points to graphing lines by making a table of solutions to an equation. Properties of vertical and horizontal lines and intercepts are included. Graphing linear equations at the end of the course gives students a good opportunity to review evaluating expressions and solving equations.

All chapters are broken down into multiple sections, the titles of which can be viewed in the Table of Contents.

Accuracy of Content

We have taken great pains to ensure the validity and accuracy of this text. Each chapter's manuscript underwent rounds of review and revision by a panel of active instructors. Then, prior to publication, a separate team of experts checked all text, examples, and graphics for mathematical accuracy. A third team of experts was responsible for the accuracy of the Answer Key, dutifully re-working every solution to eradicate any lingering errors. Finally, the editorial team conducted a multi-round post-production review to ensure the integrity of the content in its final form.

Pedagogical Foundation and Features

Learning Objectives

Each chapter is divided into multiple sections (or modules), each of which is organized around a set of learning objectives. The learning objectives are listed explicitly at the beginning of each section and are the focal point of every instructional element.

Narrative text

Narrative text is used to introduce key concepts, terms, and definitions, to provide real-world context, and to provide transitions between topics and examples. An informal voice was used to make the content accessible to students.

Throughout this book, we rely on a few basic conventions to highlight the most important ideas:

> Key terms are boldfaced, typically when first introduced and/or when formally defined.
> Key concepts and definitions are called out in a blue box for easy reference.

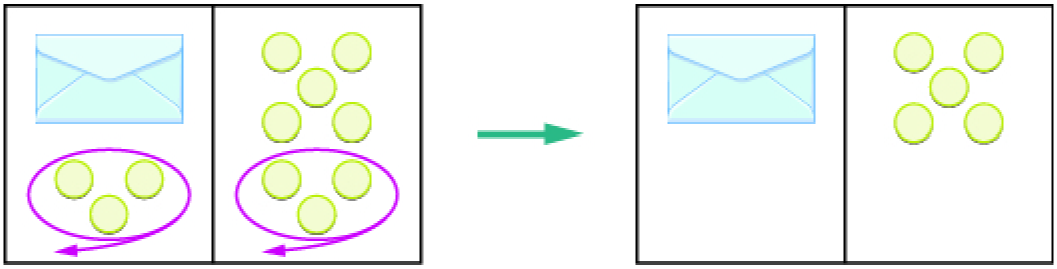
Examples

Each learning objective is supported by one or more worked examples, which demonstrate the problem-solving approaches that students must master. Typically, we include multiple Examples for each learning objective in order to model different approaches to the same type of problem, or to introduce similar problems of increasing complexity.

All Examples follow a simple two- or three-part format. First, we pose a problem or question. Next, we demonstrate the Solution, spelling out the steps along the way. Finally (for select Examples), we show students how to check the solution. Most examples are written in a two-column format, with explanation on the left and math on the right to mimic the way that instructors “talk through” examples as they write on the board in class.

Figures

Prealgebra contains many figures and illustrations. Art throughout the text adheres to a clear, understated style, drawing the eye to the most important information in each figure while minimizing visual distractions.



Supporting Features

Four small but important features serve to support Examples:

Be Prepared!

Each section, beginning with Section 1.2, starts with a few “Be Prepared!” exercises so that students can determine if they have mastered the prerequisite skills for the section. Reference is made to specific Examples from previous sections so students who need further review can easily find explanations. Answers to these exercises can be found in the supplemental resources that accompany this title.

How To

A “How To” is a list of steps necessary to solve a certain type of problem. A "How To" typically precedes an Example.

Try It

A “Try It” exercise immediately follows an Example, providing the student with an immediate opportunity to solve a similar problem. In the web view version of the text, students can click an Answer link directly below the question to check their understanding. In the PDF, answers to the Try It exercises are located in the Answer Key.

Media

▶

The “Media” icon appears at the conclusion of each section, just prior to the Section Exercises. This icon marks a list of links to online video tutorials that reinforce the concepts and skills introduced in the section.

Disclaimer: While we have selected tutorials that closely align to our learning objectives, we did not produce these tutorials, nor were they specifically produced or tailored to accompany *Prealgebra*.

Section Exercises

Each section of every chapter concludes with a well-rounded set of exercises that can be assigned as homework or used selectively for guided practice. Exercise sets are named Practice Makes Perfect to encourage completion of homework assignments.

- Exercises correlate to the learning objectives. This facilitates assignment of personalized study plans based on individual student needs.
- Exercises are carefully sequenced to promote building of skills.
- Values for constants and coefficients were chosen to practice and reinforce arithmetic facts.
- Even and odd-numbered exercises are paired.
- Exercises parallel and extend the text examples and use the same instructions as the examples to help students easily recognize the connection.
- Applications are drawn from many everyday experiences, as well as those traditionally found in college math texts.
- **Everyday Math** highlights practical situations using the math concepts from that particular section.
- **Writing Exercises** are included in every Exercise Set to encourage conceptual understanding, critical thinking, and literacy.

Chapter Review Features

The end of each chapter includes a review of the most important takeaways, as well as additional practice problems that students can use to prepare for exams.

> **Key Terms** provides a formal definition for each bold-faced term in the chapter.
> **Key Concepts** summarizes the most important ideas introduced in each section, linking back to the relevant Example(s) in case students need to review.
> **Chapter Review Exercises** includes practice problems that recall the most important concepts from each section.
> **Practice Test** includes additional problems assessing the most important learning objectives from the chapter.
> **Answer Key** includes the answers to all Try It exercises and every other exercise from the Section Exercises, Chapter Review Exercises, and Practice Test.

Additional Resources

Student and Instructor Resources

We’ve compiled additional resources for both students and instructors, including Getting Started Guides, manipulative mathematics worksheets, Links to Literacy assignments, and an answer key to Be Prepared Exercises. Instructor resources require a verified instructor account, which can be requested on your openstax.org log-in. Take advantage of these resources to supplement your OpenStax book.

Partner Resources

OpenStax Partners are our allies in the mission to make high-quality learning materials affordable and accessible to students and instructors everywhere. Their tools integrate seamlessly with our OpenStax titles at a low cost. To access the partner resources for your text, visit your book page on openstax.org.

About the Authors

Senior Contributing Authors

Lynn Marecek and MaryAnne Anthony-Smith have been teaching mathematics at Santa Ana College for many years and have worked together on several projects aimed at improving student learning in developmental math courses. They are the authors of *Strategies for Success: Study Skills for the College Math Student*, published by Pearson HigherEd.

Lynn Marecek, Santa Ana College
Lynn Marecek has focused her career on meeting the needs of developmental math students. At Santa Ana College she has been awarded the Distinguished Faculty Award, Innovation Award, and the Curriculum Development Award four times. She is a Coordinator of Freshman Experience Program, the Department Facilitator for Redesign, and a member of the Student Success and Equity Committee and the Basic Skills Initiative Task Force. Lynn holds a bachelor's degree from Valparaiso University and master's degrees from Purdue University and National University.

MaryAnne Anthony-Smith, Santa Ana College
MaryAnne has served as department chair, acting dean, chair of the professional development committee, institutional researcher, and faculty coordinator on several state and federally-funded grants and was a member of AMATYC's Placement and Assessment Committee. She is the community college coordinator of California's Mathematics Diagnostic Testing Project.

Reviewers

Tony Ayers, Collin College Preston Ridge Campus
David Behrman, Somerset Community College
Brandie Biddy, Cecil College
Bryan Blount, Kentucky Wesleyan College
Steven Boettcher, Estrella Mountain Community College
Kimberlyn Brooks, Cuyahoga Community College
Pamela Burleson, Lone Star College University Park
Tamara Carter, Texas A&M University
Phil Clark, Scottsdale Community College
Christina Cornejo, Erie Community College
Denise Cutler, Bay de Noc Community College
Richard Darnell, Eastern Wyoming College
Robert Diaz, Fullerton College
Karen Dillon, Thomas Nelson Community College
Valeree Falduto, Palm Beach State
Bryan Faulkner, Ferrum College
David French, Tidewater Community College
Stephanie Gable, Columbus State University
Heather Gallacher, Cleveland State University
Rachel Gross, Towson University
Dianne Hendrickson, Becker College
Linda Hunt, Shawnee State University
Betty Ivory, Cuyahoga Community College
Joanne Kendall, Lone Star College System
Kevin Kennedy, Athens Technical College
Stephanie Krehl, Mid-South Community College
Allyn Leon, Imperial Valley College
Gerald LePage, Bristol Community College
Laurie Lindstrom, Bay de Noc Community College
Jonathan Lopez, Niagara University
Yixia Lu, South Suburban College
Mikal McDowell, Cedar Valley College
Kim McHale, Columbia College of Missouri
Allen Miller, Northeast Lakeview College
Michelle Moravec, Baylor University TX/McLennan Community College
Jennifer Nohai-Seaman, Housatonic Community College
Rick Norwood, East Tennessee State University

Linda Padilla, Joliet Junior College
Kelly Proffitt, Patrick Henry Community College
Teresa Richards, Butte-Glenn Community College
Christian Roldan-Johnson, College of Lake County Community College
Patricia C. Rome, Delgado Community College, City Park Campus
Kegan Samuel, Naugatuck Valley Community College
Bruny Santiago, Tarrant College Southeast Campus
Sutandra Sarkar, Georgia State University
Richard Sgarlotti, Bay Mills Community College
Chuang Shao, Rose State College
Carla VanDeSande, Arizona State University
Shannon Vinson, Wake Technical Community College
Maryam Vulis, Norwalk Community College
Toby Wagner, Chemeketa Community College
Libby Watts, Tidewater Community College
Becky Wheelock, San Diego City College

Introduction class="introduction"

Purchasing pounds of fruit at a fruit market requires a basic understanding of numbers. (credit: Dr. Karl-Heinz Hochhaus, Wikimedia Commons)

Even though counting is first taught at a young age, mastering mathematics, which is the study of numbers, requires constant attention. If it has been a while since you have studied math, it can be helpful to review basic topics. In this chapter, we will focus on numbers used for counting as well as four arithmetic operations—addition, subtraction, multiplication, and division.

We will also discuss some vocabulary that we will use throughout this book.

Introduction to Whole Numbers
By the end of this section, you will be able to:

- Identify counting numbers and whole numbers
- Model whole numbers
- Identify the place value of a digit
- Use place value to name whole numbers
- Use place value to write whole numbers
- Round whole numbers

Identify Counting Numbers and Whole Numbers

Learning algebra is similar to learning a language. You start with a basic vocabulary and then add to it as you go along. You need to practice often until the vocabulary becomes easy to you. The more you use the vocabulary, the more familiar it becomes.

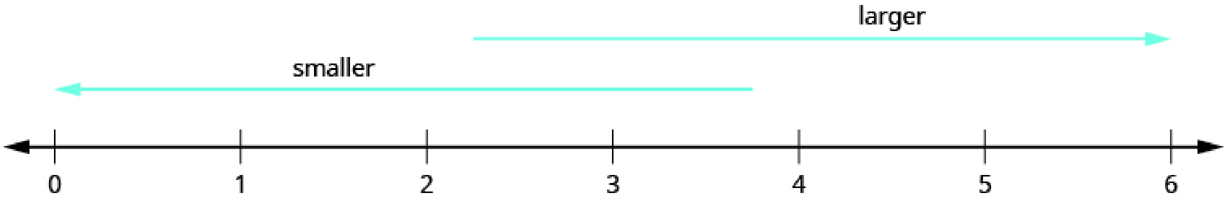
Algebra uses numbers and symbols to represent words and ideas. Let’s look at the numbers first. The most basic numbers used in algebra are those we use to count objects: $1, 2, 3, 4, 5, \ldots$ and so on. These are called the **counting numbers**. The notation “…” is called an ellipsis, which is another way to show “and so on”, or that the pattern continues endlessly. Counting numbers are also called natural numbers.

Note: Doing the Manipulative Mathematics activity Number Line-Part 1 will help you develop a better understanding of the counting numbers and the whole numbers.

Note:
Counting Numbers
The counting numbers start with 1 and continue.
Equation:

$$1, 2, 3, 4, 5 \ldots$$

Counting numbers and whole numbers can be visualized on a **number line** as shown in [link].



The numbers on the number line increase from left to right, and decrease from right to left.

The point labeled 0 is called the **origin**. The points are equally spaced to the right of 0 and labeled with the counting numbers. When a number is paired with a point, it is called the **coordinate** of the point.

The discovery of the number zero was a big step in the history of mathematics. Including zero with the counting numbers gives a new set of numbers called the **whole numbers**.

Note:
Whole Numbers
The whole numbers are the counting numbers and zero.
Equation:

$$0, 1, 2, 3, 4, 5\ldots$$

We stopped at 5 when listing the first few counting numbers and whole numbers. We could have written more numbers if they were needed to make the patterns clear.

Example:
Exercise:

Problem:

Which of the following are ⓐ counting numbers? ⓑ whole numbers?

$0, \frac{1}{4}, 3, 5.2, 15, 105$

Solution:
Solution

- ⓐ The counting numbers start at 1, so 0 is not a counting number. The numbers $3, 15,$ and 105 are all counting numbers.
- ⓑ Whole numbers are counting numbers and 0. The numbers $0, 3, 15,$ and 105 are whole numbers.

The numbers $\frac{1}{4}$ and 5.2 are neither counting numbers nor whole numbers. We will discuss these numbers later.

Note:
Exercise:

Problem:

Which of the following are ⓐ counting numbers ⓑ whole numbers?

$0, \frac{2}{3}, 2, 9, 11.8, 241, 376$

Solution:

- ⓐ 2, 9, 241, 376
- ⓑ 0, 2, 9, 241, 376

Note:
Exercise:

Problem:

Which of the following are ⓐ counting numbers ⓑ whole numbers?

$0, \frac{5}{3}, 7, 8.8, 13, 201$

Solution:

- ⓐ 7, 13, 201
- ⓑ 0, 7, 13, 201

Model Whole Numbers

Our number system is called a **place value system** because the value of a digit depends on its position, or place, in a number. The number 537 has a different value than the number 735. Even though they use the same digits, their value is different because of the different placement of the 3 and the 7 and the 5.

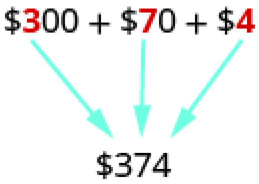
Money gives us a familiar model of place value. Suppose a wallet contains three $100 bills, seven $10 bills, and four $1 bills. The amounts are summarized in [link]. How much money is in the wallet?

Three \$100 bills
3 × \$100
\$300

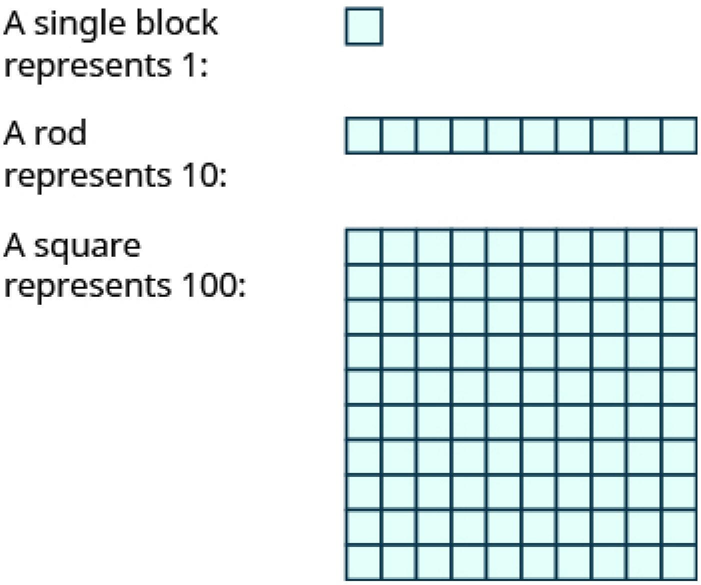
Seven \$10 bills
7 × \$10
\$70

Four \$1 bills
4 × \$1
\$4

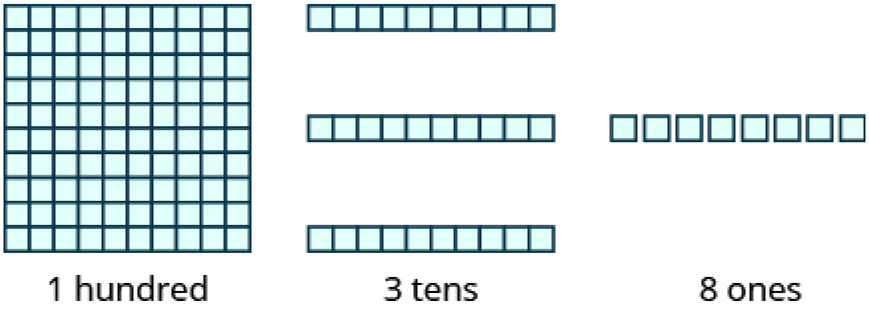
Find the total value of each kind of bill, and then add to find the total. The wallet contains \$374.



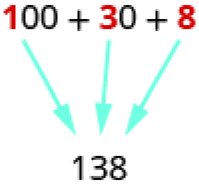
Base-10 blocks provide another way to model place value, as shown in [link]. The blocks can be used to represent hundreds, tens, and ones. Notice that the tens rod is made up of 10 ones, and the hundreds square is made of 10 tens, or 100 ones.



[link] shows the number 138 modeled with base-10 blocks.



We use place value notation to show the value of the number 138.

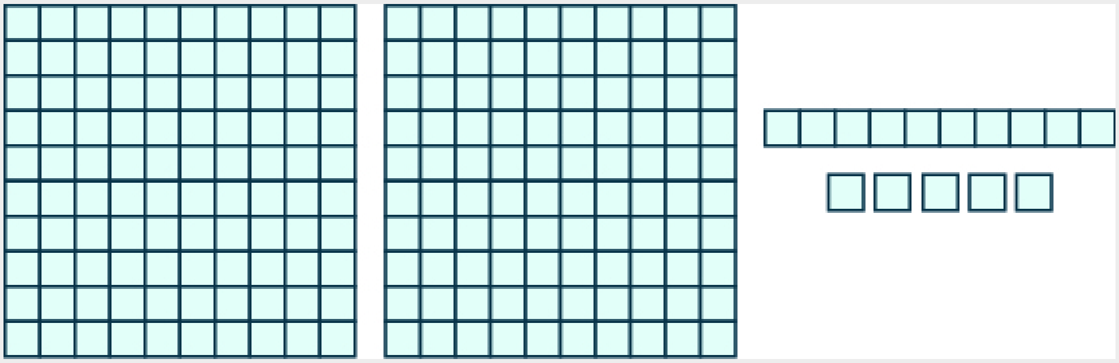


Digit	Place value	Number	Value	Total value
1	hundreds	1	100	100
3	tens	3	10	30
8	ones	8	1	$+\ 8$
				Sum $=138$

Example:
Exercise:

Problem:

Use place value notation to find the value of the number modeled by the base-10 blocks shown.

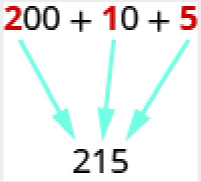


Solution:
Solution

There are 2 hundreds squares, which is 200.

There is 1 tens rod, which is 10.

There are 5 ones blocks, which is 5.



Digit	Place value	Number	Value	Total value

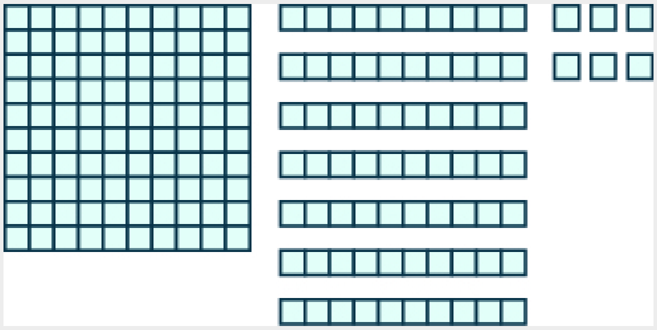
Digit	Place value	Number	Value	Total value
2	hundreds	2	100	200
1	tens	1	10	10
5	ones	5	1	+ 5
				215

The base-10 blocks model the number 215.

Note:
Exercise:

Problem:

Use place value notation to find the value of the number modeled by the base-10 blocks shown.



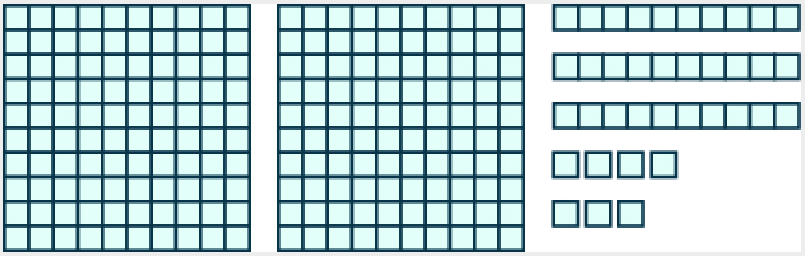
Solution:

176

Note:
Exercise:

Problem:

Use place value notation to find the value of the number modeled by the base-10 blocks shown.



Solution:

237

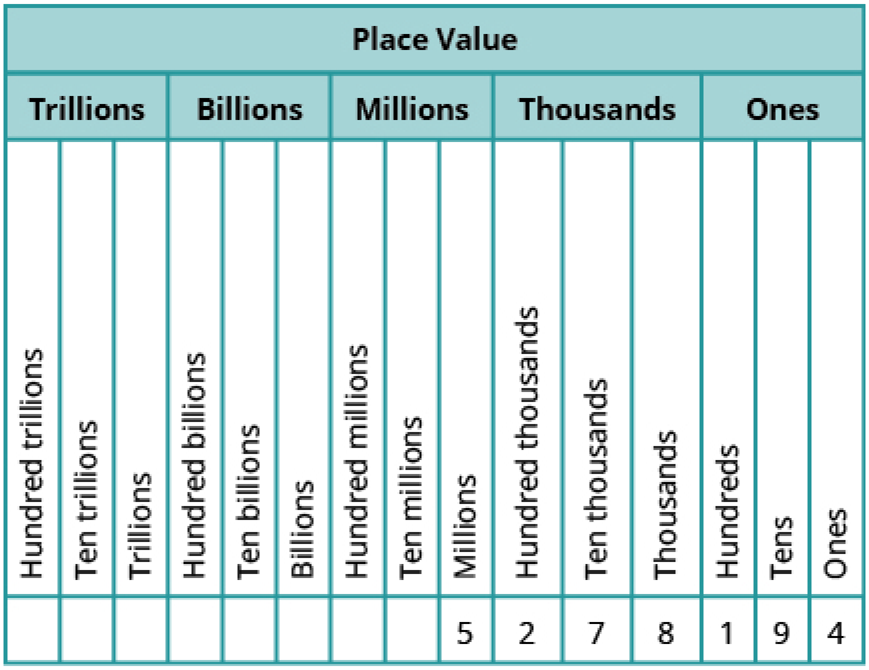
Note: Doing the Manipulative Mathematics activity “Model Whole Numbers” will help you develop a better understanding of place value of whole numbers.

Identify the Place Value of a Digit

By looking at money and base-10 blocks, we saw that each place in a number has a different value. A place value chart is a useful way to summarize this information. The place values are separated into groups of three, called periods. The periods are *ones*, *thousands*, *millions*, *billions*, *trillions*, and so on. In a written number, commas separate the periods.

Just as with the base-10 blocks, where the value of the tens rod is ten times the value of the ones block and the value of the hundreds square is ten times the tens rod, the value of each place in the place-value chart is ten times the value of the place to the right of it.

[link] shows how the number 5,278,194 is written in a place value chart.



Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

- The digit 5 is in the millions place. Its value is 5,000,000.
- The digit 2 is in the hundred thousands place. Its value is 200,000.
- The digit 7 is in the ten thousands place. Its value is 70,000.
- The digit 8 is in the thousands place. Its value is 8,000.
- The digit 1 is in the hundreds place. Its value is 100.
- The digit 9 is in the tens place. Its value is 90.
- The digit 4 is in the ones place. Its value is 4.

Example:
Exercise:

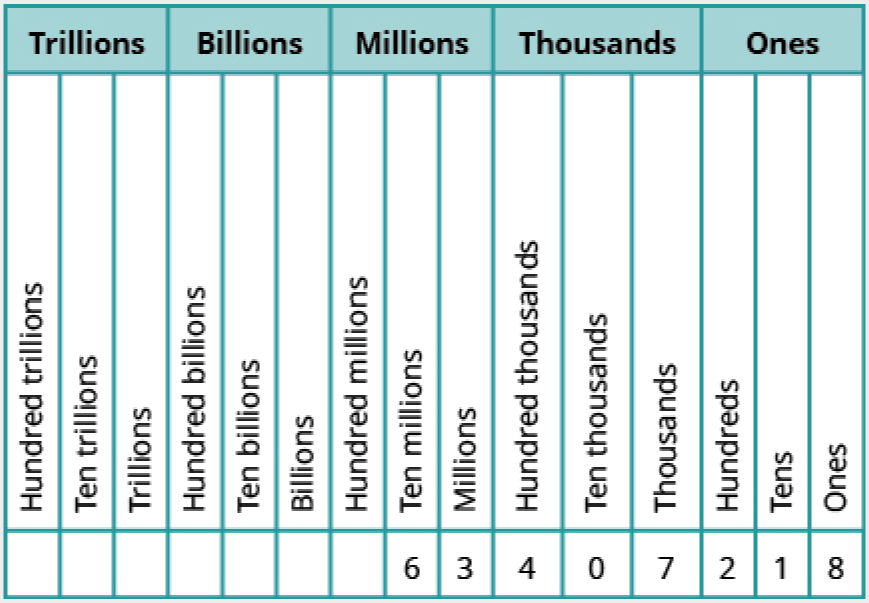
Problem:

In the number 63,407,218; find the place value of each of the following digits:

ⓐ 7
ⓑ 0
ⓒ 1
ⓓ 6
ⓔ 3

Solution:
Solution

Write the number in a place value chart, starting at the right.



Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
							6	3	4	0	7	2	1	8

- ⓐ The 7 is in the thousands place.
- ⓑ The 0 is in the ten thousands place.
- ⓒ The 1 is in the tens place.
- ⓓ The 6 is in the ten millions place.
- ⓔ The 3 is in the millions place.

Note:
Exercise:

Problem:

For each number, find the place value of digits listed: 27,493,615

ⓐ 2
ⓑ 1
ⓒ 4
ⓓ 7
ⓔ 5

Solution:

ⓐ ten millions
ⓑ tens
ⓒ hundred thousands
ⓓ millions
ⓔ ones

Note:
Exercise:

Problem:

For each number, find the place value of digits listed: 519,711,641,328

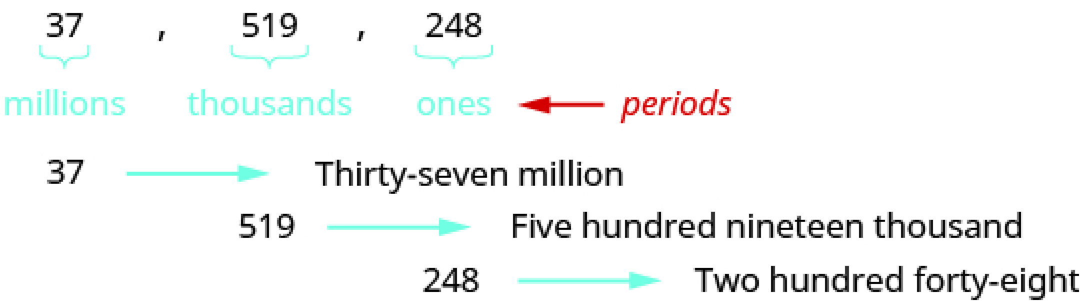
ⓐ 9
ⓑ 4
ⓒ 2
ⓓ 6
ⓔ 7

Solution:

ⓐ billions
ⓑ ten thousands
ⓒ tens
ⓓ hundred thousands
ⓔ hundred millions

Use Place Value to Name Whole Numbers

When you write a check, you write out the number in words as well as in digits. To write a number in words, write the number in each period followed by the name of the period without the ‘s’ at the end. Start with the digit at the left, which has the largest place value. The commas separate the periods, so wherever there is a comma in the number, write a comma between the words. The ones period, which has the smallest place value, is not named.



So the number 37,519,248 is written thirty-seven million, five hundred nineteen thousand, two hundred forty-eight.

Notice that the word *and* is not used when naming a whole number.

Note:
Name a whole number in words.

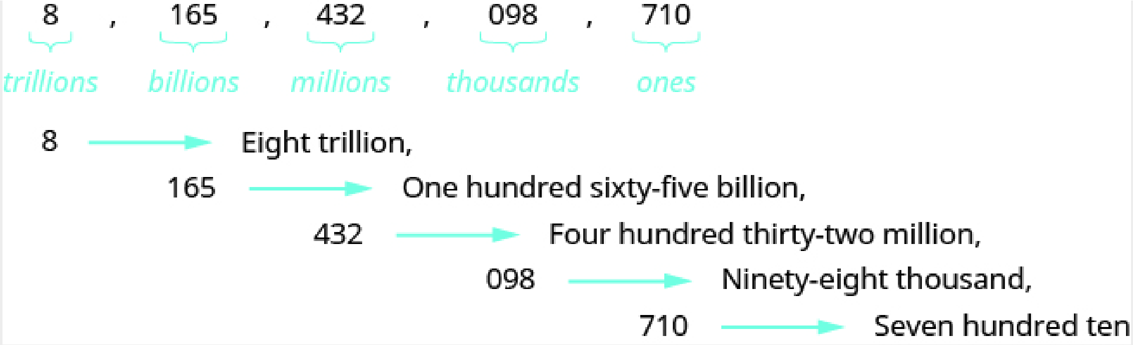
Starting at the digit on the left, name the number in each period, followed by the period name. Do not include the period name for the ones.
Use commas in the number to separate the periods.

Example:
Exercise:

Problem: Name the number 8,165,432,098,710 in words.

Solution:
Solution

Begin with the leftmost digit, which is 8. It is in the trillions place.	eight trillion
The next period to the right is billions.	one hundred sixty-five billion
The next period to the right is millions.	four hundred thirty-two million
The next period to the right is thousands.	ninety-eight thousand
The rightmost period shows the ones.	seven hundred ten



Putting all of the words together, we write 8,165,432,098,710 as eight trillion, one hundred sixty-five billion, four hundred thirty-two million, ninety-eight thousand, seven hundred ten.

Note:
Exercise:

Problem: Name each number in words: 9,258,137,904,061

Solution:

nine trillion, two hundred fifty-eight billion, one hundred thirty-seven million, nine hundred four thousand, sixty-one

Note:
Exercise:

Problem: Name each number in words: 17,864,325,619,004

Solution:

seventeen trillion, eight hundred sixty-four billion, three hundred twenty-five million, six hundred nineteen thousand, four

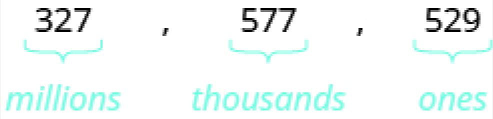
Example:
Exercise:

Problem:

A student conducted research and found that the number of mobile phone users in the United States during one month in 2014 was 327,577,529. Name that number in words.

Solution:
Solution

Identify the periods associated with the number.



Name the number in each period, followed by the period name. Put the commas in to separate the periods.

Millions period: three hundred twenty-seven million

Thousands period: five hundred seventy-seven thousand

Ones period: five hundred twenty-nine

So the number of mobile phone users in the Unites States during the month of April was three hundred twenty-seven million, five hundred seventy-seven thousand, five hundred twenty-nine.

Note:
Exercise:

Problem:

The population in a country is 316,128,839. Name that number.

Solution:

three hundred sixteen million, one hundred twenty-eight thousand, eight hundred thirty nine

Note:
Exercise:

Problem: One year is 31,536,000 seconds. Name that number.

Solution:

thirty one million, five hundred thirty-six thousand

Use Place Value to Write Whole Numbers

We will now reverse the process and write a number given in words as digits.

Note:
Use place value to write a whole number.

Identify the words that indicate periods. (Remember the ones period is never named.)
Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.
Name the number in each period and place the digits in the correct place value position.

Example:
Exercise:

Problem: Write the following numbers using digits.

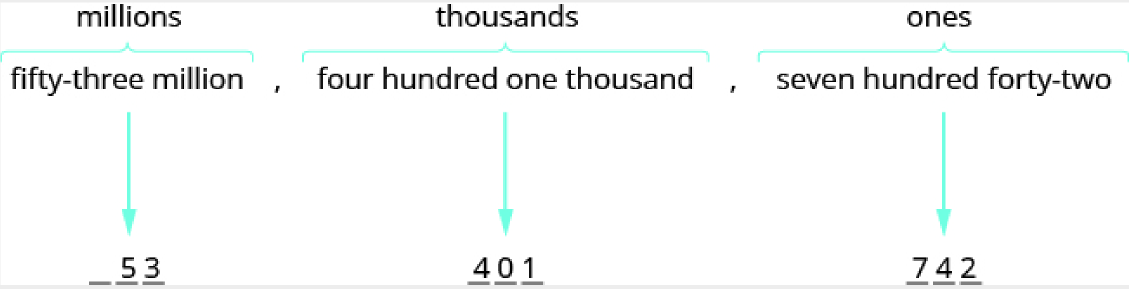
- ⓐ fifty-three million, four hundred one thousand, seven hundred forty-two
- ⓑ nine billion, two hundred forty-six million, seventy-three thousand, one hundred eighty-nine

Solution:
Solution

ⓐ Identify the words that indicate periods.

Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.

Then write the digits in each period.

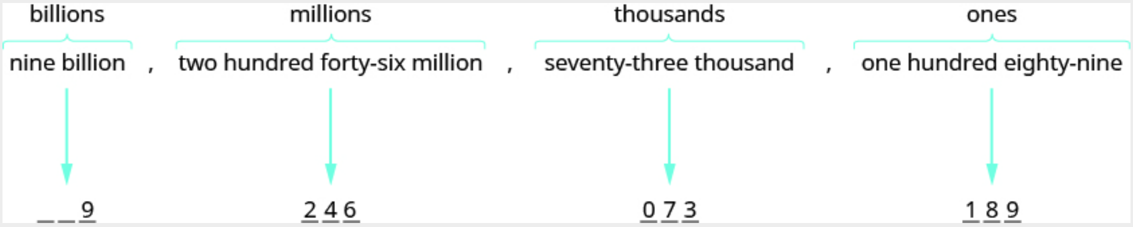


Put the numbers together, including the commas. The number is 53,401,742.

ⓑ Identify the words that indicate periods.

Except for the first period, all other periods must have three places. Draw three blanks to indicate the number of places needed in each period. Separate the periods by commas.

Then write the digits in each period.



The number is 9,246,073,189.

Notice that in part ⓑ, a zero was needed as a place-holder in the hundred thousands place. Be sure to write zeros as needed to make sure that each period, except possibly the first, has three places.

Note:
Exercise:

Problem: Write each number in standard form:

fifty-three million, eight hundred nine thousand, fifty-one.

Solution:

53,809,051

Note:
Exercise:

Problem: Write each number in standard form:

two billion, twenty-two million, seven hundred fourteen thousand, four hundred sixty-six.

Solution:

2,022,714,466

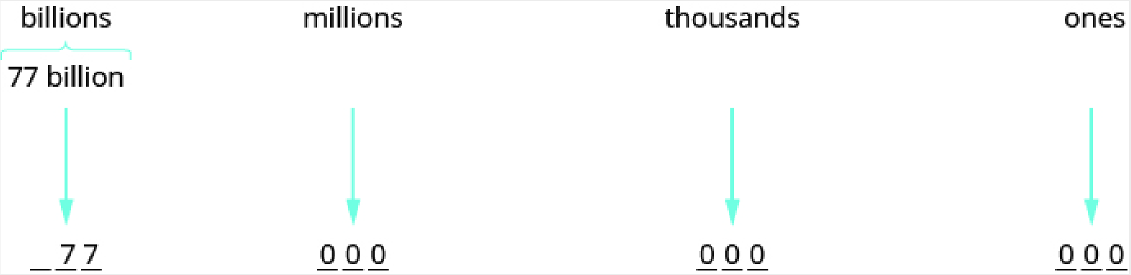
Example:
Exercise:

Problem:

A state budget was about $77 billion. Write the budget in standard form.

Solution:
Solution

Identify the periods. In this case, only two digits are given and they are in the billions period. To write the entire number, write zeros for all of the other periods.



So the budget was about $77,000,000,000.

Note:
Exercise:

Problem: Write each number in standard form:

The closest distance from Earth to Mars is about 34 million miles.

Solution:

34,000,000 miles

Note:
Exercise:

Problem: Write each number in standard form:

The total weight of an aircraft carrier is 204 million pounds.

Solution:

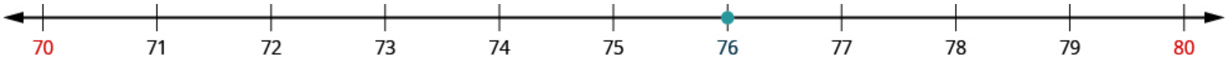
204,000,000 pounds

Round Whole Numbers

In 2013, the U.S. Census Bureau reported the population of the state of New York as 19,651,127 people. It might be enough to say that the population is approximately 20 million. The word *approximately* means that 20 million is not the exact population, but is close to the exact value.

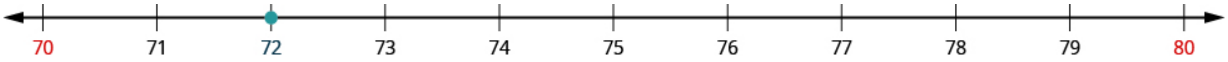
The process of approximating a number is called **rounding**. Numbers are rounded to a specific place value depending on how much accuracy is needed. 20 million was achieved by rounding to the millions place. Had we rounded to the one hundred thousands place, we would have 19,700,000 as a result. Had we rounded to the ten thousands place, we would have 19,650,000 as a result, and so on. The place value to which we round to depends on how we need to use the number.

Using the number line can help you visualize and understand the rounding process. Look at the number line in [link]. Suppose we want to round the number 76 to the nearest ten. Is 76 closer to 70 or 80 on the number line?



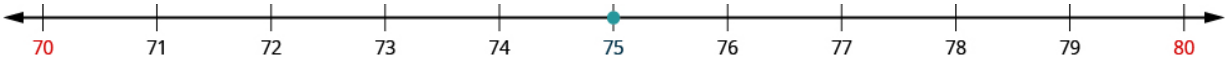
We can see that 76 is closer to 80 than to 70. So 76 rounded to the nearest ten is 80.

Now consider the number 72. Find 72 in [link].



We can see that 72 is closer to 70, so 72 rounded to the nearest ten is 70.

How do we round 75 to the nearest ten. Find 75 in [link].

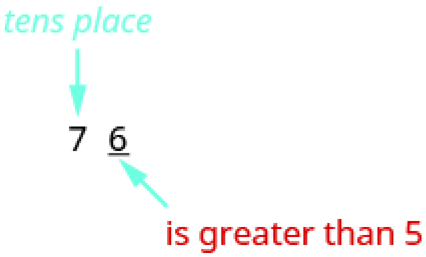


The number 75 is exactly midway between 70 and 80.

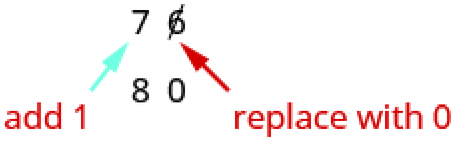
So that everyone rounds the same way in cases like this, mathematicians have agreed to round to the higher number, 80. So, 75 rounded to the nearest ten is 80.

Now that we have looked at this process on the number line, we can introduce a more general procedure. To round a number to a specific place, look at the number to the right of that place. If the number is less than 5, round down. If it is greater than or equal to 5, round up.

So, for example, to round 76 to the nearest ten, we look at the digit in the ones place.

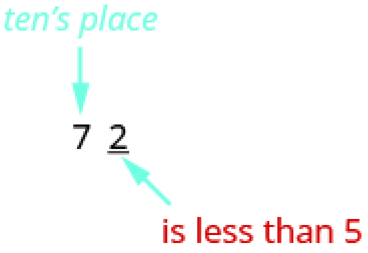


The digit in the ones place is a 6. Because 6 is greater than or equal to 5, we increase the digit in the tens place by one. So the 7 in the tens place becomes an 8. Now, replace any digits to the right of the 8 with zeros. So, 76 rounds to 80.

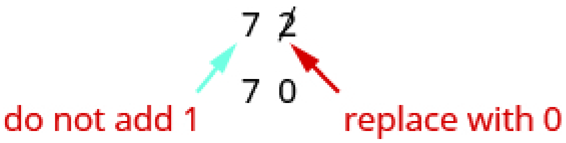


76 rounded to the nearest ten is 80.

Let’s look again at rounding 72 to the nearest 10. Again, we look to the ones place.



The digit in the ones place is 2. Because 2 is less than 5, we keep the digit in the tens place the same and replace the digits to the right of it with zero. So 72 rounded to the nearest ten is 70.



Note:
Round a whole number to a specific place value.

Locate the given place value. All digits to the left of that place value do not change.
Underline the digit to the right of the given place value.
Determine if this digit is greater than or equal to 5.

- Yes—add 1 to the digit in the given place value.
- No—do not change the digit in the given place value.

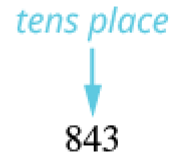
Replace all digits to the right of the given place value with zeros.

Example:
Exercise:

Problem: Round 843 to the nearest ten.

Solution:
Solution

Locate the tens place.	

	
Underline the digit to the right of the tens place.	$84\underline{3}$
Since 3 is less than 5, do not change the digit in the tens place.	$84\underline{3}$
Replace all digits to the right of the tens place with zeros.	$84\underline{0}$
	Rounding 843 to the nearest ten gives 840.

Note:
Exercise:

Problem: Round to the nearest ten: 157.

Solution:

160

Note:
Exercise:

Problem: Round to the nearest ten: 884.

Solution:

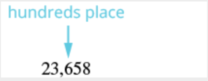
880

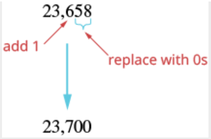
Example:
Exercise:

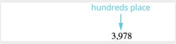
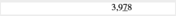
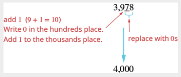
Problem: Round each number to the nearest hundred:

ⓐ 23,658
ⓑ 3,978

Solution:
Solution

ⓐ	
Locate the hundreds place.	
The digit of the right of the hundreds place is 5. Underline the digit to the right of the hundreds place.	$23{,}6\underline{5}8$

Since 5 is greater than or equal to 5, round up by adding 1 to the digit in the hundreds place. Then replace all digits to the right of the hundreds place with zeros.	 So 23,658 rounded to the nearest hundred is 23,700.

ⓑ	
Locate the hundreds place.	
Underline the digit to the right of the hundreds place.	
The digit to the right of the hundreds place is 7. Since 7 is greater than or equal to 5, round up by added 1 to the 9. Then place all digits to the right of the hundreds place with zeros.	 3,978 add 1 (9 + 1 = 10) Write 0 in the hundreds place. Add 1 to the thousands place. replace with 0s 4,000 So 3,978 rounded to the nearest hundred is 4,000.

Note:
Exercise:

Problem: Round to the nearest hundred: 17,852.

Solution:

17,900

Note:
Exercise:

Problem: Round to the nearest hundred: 4,951.

Solution:

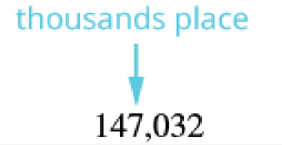
5,000

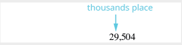
Example:
Exercise:

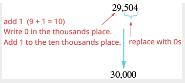
Problem: Round each number to the nearest thousand:

ⓐ 147,032
ⓑ 29,504

Solution:
Solution

ⓐ	
Locate the thousands place. Underline the digit to the right of the thousands place.	
The digit to the right of the thousands place is 0. Since 0 is less than 5, we do not change the digit in the thousands place.	$147,\underline{0}32$
We then replace all digits to the right of the thousands pace with zeros.	$147,\underline{0}32$ So 147,032 rounded to the nearest thousand is 147,000.

ⓑ	
Locate the thousands place.	
Underline the digit to the right of the thousands place.	$29,\underline{5}04$
The digit to the right of the thousands place is 5.	

Since 5 is greater than or equal to 5, round up by adding 1 to the 9. Then replace all digits to the right of the thousands place with zeros.	 29,504 add 1 (9 + 1 = 10) Write 0 in the thousands place. Add 1 to the ten thousands place. replace with 0s 30,000 So 29,504 rounded to the nearest thousand is 30,000.

Notice that in part ⓑ, when we add 1 thousand to the 9 thousands, the total is 10 thousands. We regroup this as 1 ten thousand and 0 thousands. We add the 1 ten thousand to the 3 ten thousands and put a 0 in the thousands place.

Note:
Exercise:

Problem: Round to the nearest thousand: 63,921.

Solution:

64,000

Note:
Exercise:

Problem: Round to the nearest thousand: 156,437.

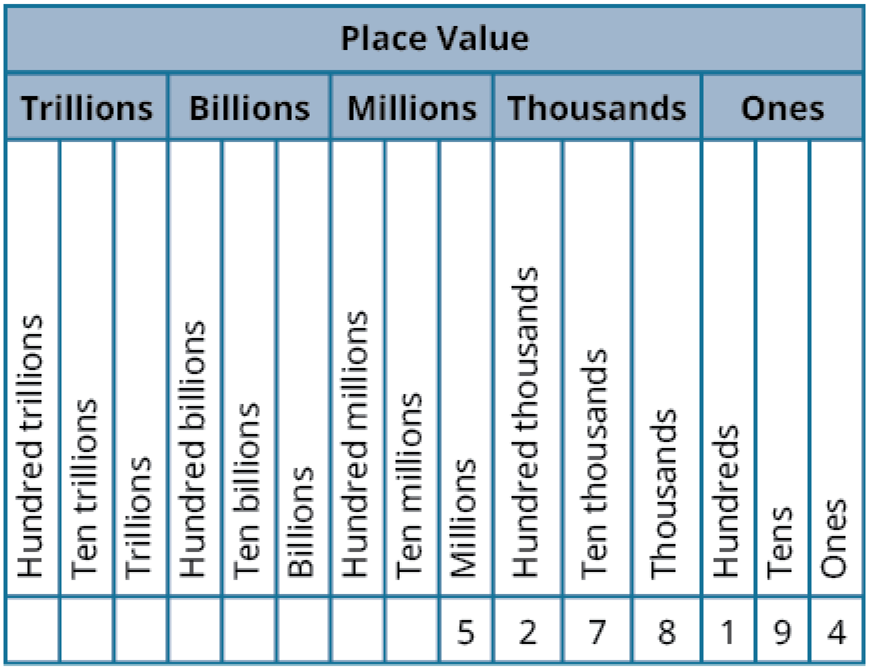
Solution:

156,000

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Determine Place Value
- Write a Whole Number in Digits from Words

Key Concepts



Place Value														
Trillions			Billions			Millions			Thousands			Ones		
Hundred trillions	Ten trillions	Trillions	Hundred billions	Ten billions	Billions	Hundred millions	Ten millions	Millions	Hundred thousands	Ten thousands	Thousands	Hundreds	Tens	Ones
								5	2	7	8	1	9	4

- **Name a whole number in words.**

  Starting at the digit on the left, name the number in each period, followed by the period name. Do not include the period name for the ones.
  Use commas in the number to separate the periods.

- **Use place value to write a whole number.**

  Identify the words that indicate periods. (Remember the ones period is

never named.)
Draw three blanks to indicate the number of places needed in each period.
Name the number in each period and place the digits in the correct place value position.

- **Round a whole number to a specific place value.**

  Locate the given place value. All digits to the left of that place value do not change.
  Underline the digit to the right of the given place value.
  Determine if this digit is greater than or equal to 5. If yes—add 1 to the digit in the given place value. If no—do not change the digit in the given place value.
  Replace all digits to the right of the given place value with zeros.

Practice Makes Perfect

Identify Counting Numbers and Whole Numbers

In the following exercises, determine which of the following numbers are ⓐ counting numbers ⓑ whole numbers.

Exercise:

Problem: $0, \frac{2}{3}, 5, 8.1, 125$

Solution:

ⓐ 5, 125
ⓑ 0, 5, 125

Exercise:

Problem: $0, \frac{7}{10}, 3, 20.5, 300$

Exercise:

Problem: $0, \frac{4}{9}, 3.9, 50, 221$

Solution:

ⓐ 50, 221
ⓑ 0, 50, 221

Exercise:

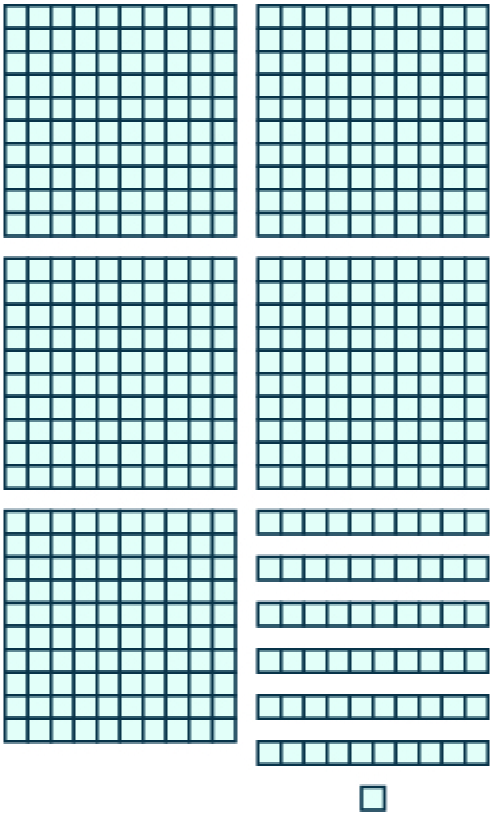
Problem: $0, \frac{3}{5}, 10, 303, 422.6$

Model Whole Numbers

In the following exercises, use place value notation to find the value of the number modeled by the base-10 blocks.

Exercise:

Problem:

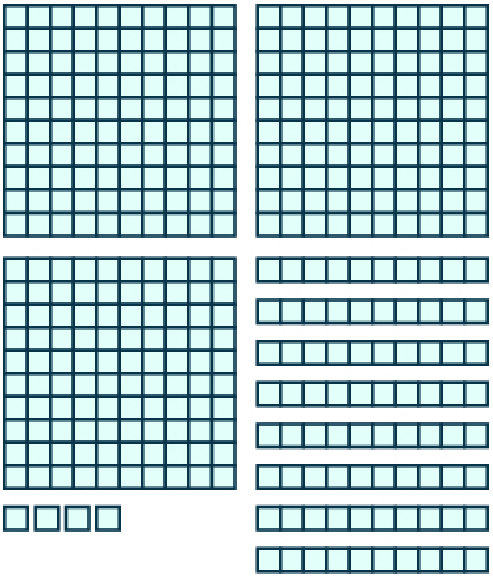


Solution:

561

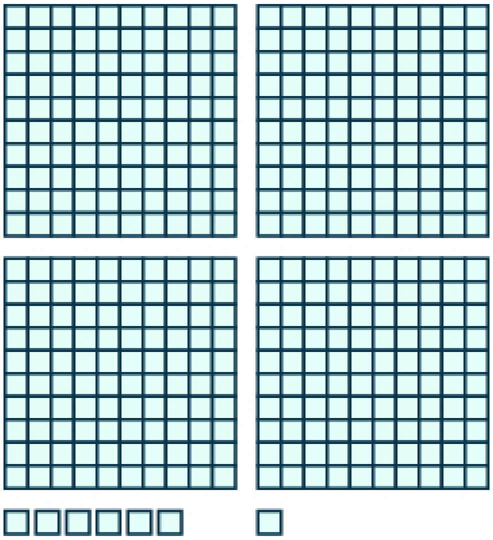
Exercise:

Problem:



Exercise:

Problem:

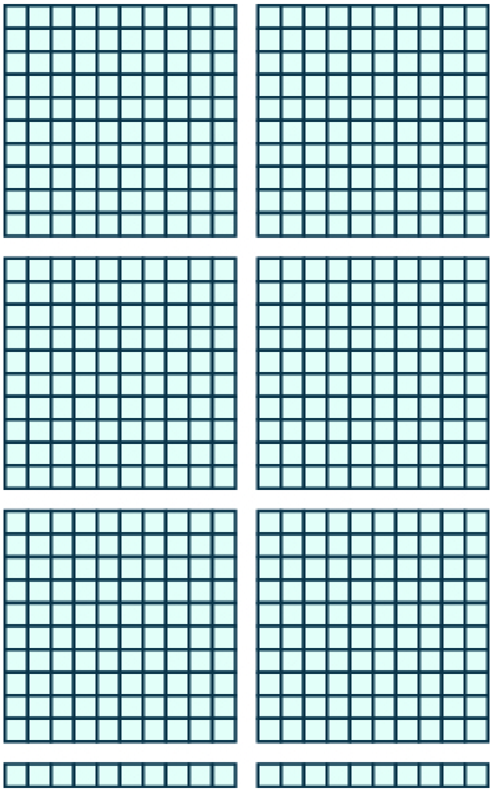


Solution:

407

Exercise:

Problem:



Identify the Place Value of a Digit

In the following exercises, find the place value of the given digits.

Exercise:

Problem: 579,601

ⓐ 9
ⓑ 6
ⓒ 0
ⓓ 7
ⓔ 5

Solution:

ⓐ thousands
ⓑ hundreds

ⓒ tens
ⓓ ten thousands
ⓔ hundred thousands

Exercise:

Problem: 398,127

ⓐ 9
ⓑ 3
ⓒ 2
ⓓ 8
ⓔ 7

Exercise:

Problem: 56,804,379

ⓐ 8
ⓑ 6
ⓒ 4
ⓓ 7
ⓔ 0

Solution:

ⓐ hundred thousands
ⓑ millions
ⓒ thousands
ⓓ tens
ⓔ ten thousands

Exercise:

Problem: 78,320,465

ⓐ 8
ⓑ 4
ⓒ 2
ⓓ 6
ⓔ 7

Use Place Value to Name Whole Numbers

In the following exercises, name each number in words.

Exercise:

Problem: 1,078

Solution:

One thousand, seventy-eight

Exercise:

Problem: 5,902

Exercise:

Problem: 364,510

Solution:

Three hundred sixty-four thousand, five hundred ten

Exercise:

Problem: 146,023

Exercise:

Problem: 5,846,103

Solution:

Five million, eight hundred forty-six thousand, one hundred three

Exercise:

Problem: 1,458,398

Exercise:

Problem: 37,889,005

Solution:

Thirty seven million, eight hundred eighty-nine thousand, five

Exercise:

Problem: 62,008,465

Exercise:

Problem: The height of Mount Ranier is 14,410 feet.

Solution:

Fourteen thousand, four hundred ten

Exercise:

Problem: The height of Mount Adams is 12,276 feet.

Exercise:

Problem: Seventy years is 613,200 hours.

Solution:

Six hundred thirteen thousand, two hundred

Exercise:

Problem: One year is 525,600 minutes.

Exercise:

Problem:

The U.S. Census estimate of the population of Miami-Dade county was 2,617,176.

Solution:

Two million, six hundred seventeen thousand, one hundred seventy-six

Exercise:

Problem: The population of Chicago was 2,718,782.

Exercise:

Problem:

There are projected to be 23,867,000 college and university students in the US in five years.

Solution:

Twenty three million, eight hundred sixty-seven thousand

Exercise:

Problem:

About twelve years ago there were 20,665,415 registered automobiles in California.

Exercise:

Problem:

The population of China is expected to reach 1,377,583,156 in 2016.

Solution:

One billion, three hundred seventy-seven million, five hundred eighty-three thousand, one hundred fifty-six

Exercise:

Problem:

The population of India is estimated at $1,267,401,849$ as of July $1, 2014$.

Use Place Value to Write Whole Numbers

In the following exercises, write each number as a whole number using digits.

Exercise:

Problem: four hundred twelve

Solution:

412

Exercise:

Problem: two hundred fifty-three

Exercise:

Problem: thirty-five thousand, nine hundred seventy-five

Solution:

35,975

Exercise:

Problem: sixty-one thousand, four hundred fifteen

Exercise:

Problem:

eleven million, forty-four thousand, one hundred sixty-seven

Solution:

11,044,167

Exercise:

Problem:

eighteen million, one hundred two thousand, seven hundred eighty-three

Exercise:

Problem:

three billion, two hundred twenty-six million, five hundred twelve thousand, seventeen

---

Solution:

3,226,512,017

Exercise:

Problem:

eleven billion, four hundred seventy-one million, thirty-six thousand, one hundred six

Exercise:

Problem:

The population of the world was estimated to be seven billion, one hundred seventy-three million people.

Solution:

7,173,000,000

Exercise:

Problem:

The age of the solar system is estimated to be four billion, five hundred sixty-eight million years.

Exercise:

Problem:

Lake Tahoe has a capacity of thirty-nine trillion gallons of water.

Solution:

39,000,000,000,000

Exercise:

Problem:

The federal government budget was three trillion, five hundred billion dollars.

Round Whole Numbers

In the following exercises, round to the indicated place value.

Exercise:

Problem: Round to the nearest ten:

ⓐ 386
ⓑ 2,931

Solution:

ⓐ 390
ⓑ 2,930

Exercise:

Problem: Round to the nearest ten:

ⓐ 792
ⓑ 5,647

Exercise:

Problem: Round to the nearest hundred:

ⓐ 13,748
ⓑ 391,794

Solution:

ⓐ 13,700
ⓑ 391,800

Exercise:

Problem: Round to the nearest hundred:

ⓐ 28,166
ⓑ 481,628

Exercise:

Problem: Round to the nearest ten:

ⓐ 1,492
ⓑ 1,497

Solution:

ⓐ 1,490
ⓑ 1,500

Exercise:

Problem: Round to the nearest thousand:

ⓐ 2,391
ⓑ 2,795

Exercise:

Problem: Round to the nearest hundred:

ⓐ 63,994
ⓑ 63,949

Solution:

ⓐ 64,000
ⓑ 63,900

Exercise:

Problem: Round to the nearest thousand:

ⓐ 163,584
ⓑ 163,246

Everyday Math

Exercise:

Problem:

Writing a Check Jorge bought a car for $\$24,493$. He paid for the car with a check. Write the purchase price in words.

Solution:

Twenty four thousand, four hundred ninety-three dollars

Exercise:

Problem:

Writing a Check Marissa’s kitchen remodeling cost $\$18,549$. She wrote a check to the contractor. Write the amount paid in words.

Exercise:

Problem:

Buying a Car Jorge bought a car for $\$24,493$. Round the price to the nearest:

ⓐ ten dollars
ⓑ hundred dollars
ⓒ thousand dollars
ⓓ ten-thousand dollars

Solution:

ⓐ $24,490
ⓑ $24,500
ⓒ $24,000
ⓓ $20,000

Exercise:

Problem:

Remodeling a Kitchen Marissa’s kitchen remodeling cost $18,549. Round the cost to the nearest:

ⓐ ten dollars
ⓑ hundred dollars
ⓒ thousand dollars
ⓓ ten-thousand dollars

Exercise:

Problem:

Population The population of China was 1,355,692,544 in 2014. Round the population to the nearest:

ⓐ billion people
ⓑ hundred-million people
ⓒ million people

Solution:

ⓐ 1,000,000,000
ⓑ 1,400,000,000
ⓒ 1,356,000,000

Exercise:

Problem:

Astronomy The average distance between Earth and the sun is 149,597,888 kilometers. Round the distance to the nearest:

ⓐ hundred-million kilometers
ⓑ ten-million kilometers
ⓒ million kilometers

Writing Exercises

Exercise:

Problem:

In your own words, explain the difference between the counting numbers and the whole numbers.

Solution:

Answers may vary. The whole numbers are the counting numbers with the inclusion of zero.

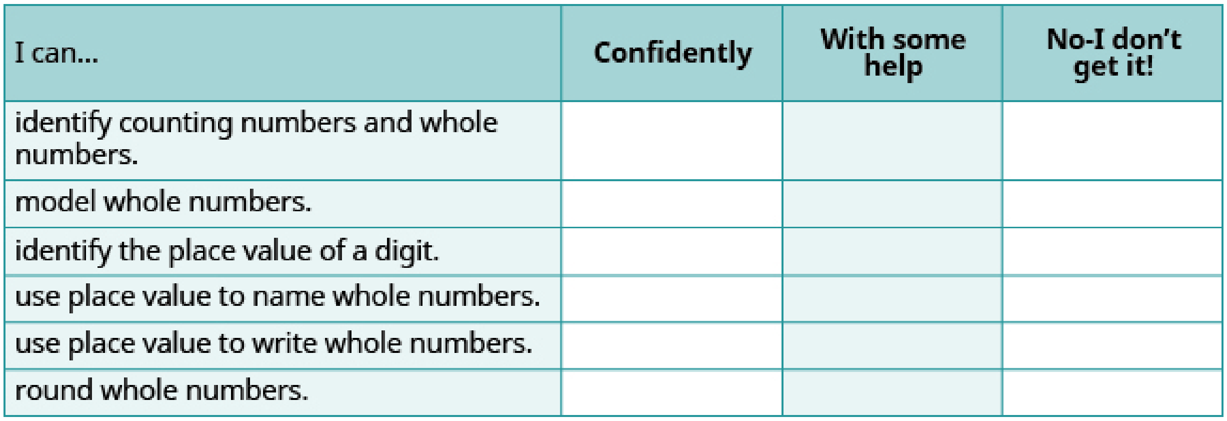
Exercise:

Problem:

Give an example from your everyday life where it helps to round numbers.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
identify counting numbers and whole numbers.			
model whole numbers.			
identify the place value of a digit.			
use place value to name whole numbers.			
use place value to write whole numbers.			
round whole numbers.			

ⓑ If most of your checks were...

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don't get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

coordinate

A number paired with a point on a number line is called the coordinate of the point.

counting numbers

The counting numbers are the numbers 1, 2, 3, ….

number line

A number line is used to visualize numbers. The numbers on the number line get larger as they go from left to right, and smaller as they go from right to left.

origin

The origin is the point labeled 0 on a number line.

place value system

Our number system is called a place value system because the value of a digit depends on its position, or place, in a number.

rounding

The process of approximating a number is called rounding.

whole numbers

The whole numbers are the numbers 0, 1, 2, 3, ….

Add Whole Numbers

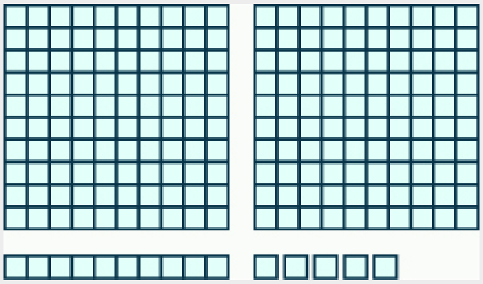
By the end of this section, you will be able to:

- Use addition notation
- Model addition of whole numbers
- Add whole numbers without models
- Translate word phrases to math notation
- Add whole numbers in applications

Note:

Before you get started, take this readiness quiz.

1. What is the number modeled by the base-10 blocks?



   If you missed this problem, review [link].
2. Write the number three hundred forty-two thousand six using digits?
   If you missed this problem, review [link].

Use Addition Notation

A college student has a part-time job. Last week he worked 3 hours on Monday and 4 hours on Friday. To find the total number of hours he worked last week, he added 3 and 4.

The operation of addition combines numbers to get a **sum**. The notation we use to find the sum of 3 and 4 is:

Equation:

$$3+4$$

We read this as *three plus four* and the result is the sum of three and four. The numbers 3 and 4 are called the addends. A math statement that includes numbers and operations is called an expression.

Note:
Addition Notation
To describe addition, we can use symbols and words.

Operation	**Notation**	**Expression**	**Read as**	**Result**
Addition	$+$	$3+4$	three plus four	the sum of 3 and 4

Example:
Exercise:

Problem: Translate from math notation to words:

ⓐ $7+1$
ⓑ $12+14$

Solution:
Solution

- ⓐ The expression consists of a plus symbol connecting the addends 7 and 1. We read this as *seven plus one*. The result is *the sum of seven and one*.

- ⓑ The expression consists of a plus symbol connecting the addends 12 and 14. We read this as *twelve plus fourteen*. The result is the *sum of twelve and fourteen*.

Note:
Exercise:

Problem: Translate from math notation to words:

ⓐ $8 + 4$
ⓑ $18 + 11$

Solution:

- ⓐ eight plus four; the sum of eight and four
- ⓑ eighteen plus eleven; the sum of eighteen and eleven

Note:
Exercise:

Problem: Translate from math notation to words:

ⓐ $21 + 16$
ⓑ $100 + 200$

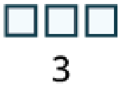
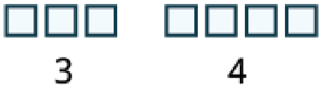
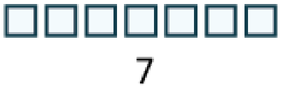
Solution:

ⓐ twenty-one plus sixteen; the sum of twenty-one and sixteen
ⓑ one hundred plus two hundred; the sum of one hundred and two hundred

Model Addition of Whole Numbers

Addition is really just counting. We will model addition with base-10 blocks. Remember, a block represents 1 and a rod represents 10. Let’s start by modeling the addition expression we just considered, $3 + 4$.

Each addend is less than 10, so we can use ones blocks.

We start by modeling the first number with 3 blocks.	
Then we model the second number with 4 blocks.	
Count the total number of blocks.	

There are 7 blocks in all. We use an equal sign $(=)$ to show the sum. A math sentence that shows that two expressions are equal is called an equation. We have shown that. $3 + 4 = 7$.

Note: Doing the Manipulative Mathematics activity “Model Addition of Whole Numbers” will help you develop a better understanding of adding whole numbers.

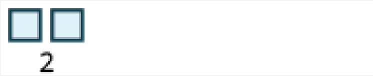
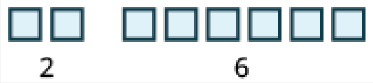
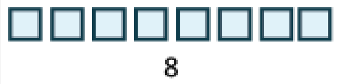
Example:
Exercise:

Problem: Model the addition $2+6$.

Solution:
Solution

$2+6$ means the sum of 2 and 6

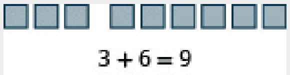
Each addend is less than 10, so we can use ones blocks.

Model the first number with 2 blocks.	
Model the second number with 6 blocks.	
Count the total number of blocks	 There are 8 blocks in all, so $2+6=8$.

Note:
Exercise:

Problem: Model: $3+6$.

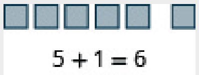
Solution:



Note:
Exercise:

Problem: Model: $5 + 1$.

Solution:



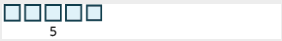
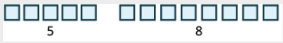
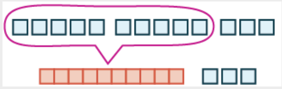
When the result is 10 or more ones blocks, we will exchange the 10 blocks for one rod.

Example:
Exercise:

Problem: Model the addition $5 + 8$.

Solution:
Solution

$5 + 8$ means the sum of 5 and 8.

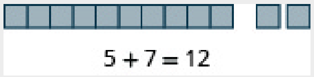
Each addend is less than 10, se we can use ones blocks.	
Model the first number with 5 blocks.	
Model the second number with 8 blocks.	
Count the result. There are more than 10 blocks so we exchange 10 ones blocks for 1 tens rod.	
Now we have 1 ten and 3 ones, which is 13.	5 + 8 = 13

Notice that we can describe the models as ones blocks and tens rods, or we can simply say *ones* and *tens*. From now on, we will use the shorter version but keep in mind that they mean the same thing.

Note:
Exercise:

Problem: Model the addition: $5 + 7$.

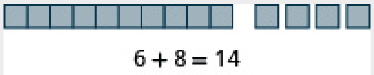
Solution:



Note:
Exercise:

Problem: Model the addition: $6 + 8$.

Solution:



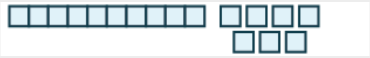
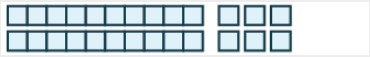
Next we will model adding two digit numbers.

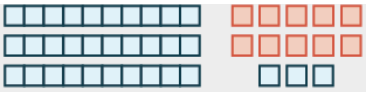
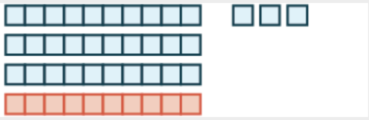
Example:
Exercise:

Problem: Model the addition: $17 + 26$.

Solution:
Solution

$17 + 26$ means the sum of 17 and 26.

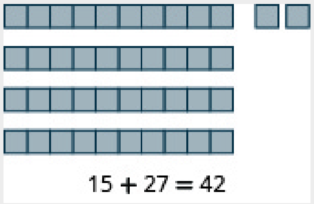
Model the 17.	1 ten and 7 ones	
Model the 26.	2 tens and 6 ones	
Combine.	3 tens and	

	13 ones	
Exchange 10 ones for 1 ten.	4 tens and 3 ones $40 + 3 = 43$	
We have shown that $17 + 26 = 43$		

Note:
Exercise:

Problem: Model each addition: $15 + 27$.

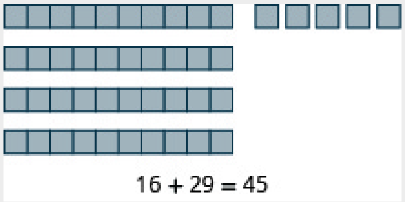
Solution:



Note:
Exercise:

Problem: Model each addition: $16 + 29$.

Solution:



Add Whole Numbers Without Models

Now that we have used models to add numbers, we can move on to adding without models. Before we do that, make sure you know all the one digit addition facts. You will need to use these number facts when you add larger numbers.

Imagine filling in [link] by adding each row number along the left side to each column number across the top. Make sure that you get each sum shown. If you have trouble, model it. It is important that you memorize any number facts you do not already know so that you can quickly and reliably use the number facts when you add larger numbers.

+	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14

+	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Did you notice what happens when you add zero to a number? The sum of any number and zero is the number itself. We call this the Identity Property of Addition. Zero is called the additive identity.

Note:
Identity Property of Addition
The sum of any number a and 0 is the number.
Equation:

$$\begin{aligned} a + 0 &= a \\ 0 + a &= a \end{aligned}$$

Example:
Exercise:

Problem: Find each sum:

ⓐ $0 + 11$
ⓑ $42 + 0$

Solution:
Solution

ⓐ The first addend is zero. The sum of any number and zero is the number.	$0+11=11$
ⓑ The second addend is zero. The sum of any number and zero is the number.	$42+0=42$

Note:
Exercise:

Problem: Find each sum:

ⓐ $0+19$
ⓑ $39+0$

Solution:

ⓐ $0+19=19$
ⓑ $39+0=39$

Note:
Exercise:

Problem: Find each sum:

ⓐ $0+24$
ⓑ $57+0$

Solution:

ⓐ $0+24=24$
ⓑ $57+0=57$

Look at the pairs of sums.

$2+3=5$	$3+2=5$
$4+7=11$	$7+4=11$
$8+9=17$	$9+8=17$

Notice that when the order of the addends is reversed, the sum does not change. This property is called the Commutative Property of Addition, which states that changing the order of the addends does not change their sum.

Note:
Commutative Property of Addition
Changing the order of the addends a and b does not change their sum.
Equation:

$$a+b=b+a$$

Example:
Exercise:

Problem: Add:

ⓐ $8+7$
ⓑ $7+8$

Solution:
Solution

ⓐ	
Add.	$8+7$
	15

ⓑ	
Add.	$7+8$
	15

Did you notice that changing the order of the addends did not change their sum? We could have immediately known the sum from part ⓑ just by recognizing that the addends were the same as in part ⓑ, but in the reverse order. As a result, both sums are the same.

Note:
Exercise:

Problem: Add: $9+7$ and $7+9$.

Solution:

$9 + 7 = 16; 7 + 9 = 16$

Note:
Exercise:

Problem: Add: $8 + 6$ and $6 + 8$.

Solution:

$8 + 6 = 14; 6 + 8 = 14$

Example:
Exercise:

Problem: Add: $28 + 61$.

Solution:
Solution

To add numbers with more than one digit, it is often easier to write the numbers vertically in columns.

Write the numbers so the ones and tens digits line up vertically.	$\begin{array}{r} 28 \\ \underline{+61} \end{array}$
Then add the digits in each place value. Add the ones: $8 + 1 = 9$ Add the tens: $2 + 6 = 8$	$\begin{array}{r} 28 \\ \underline{+61} \\ 89 \end{array}$

Note:
Exercise:

Problem: Add: $32 + 54$.

Solution:

$32 + 54 = 86$

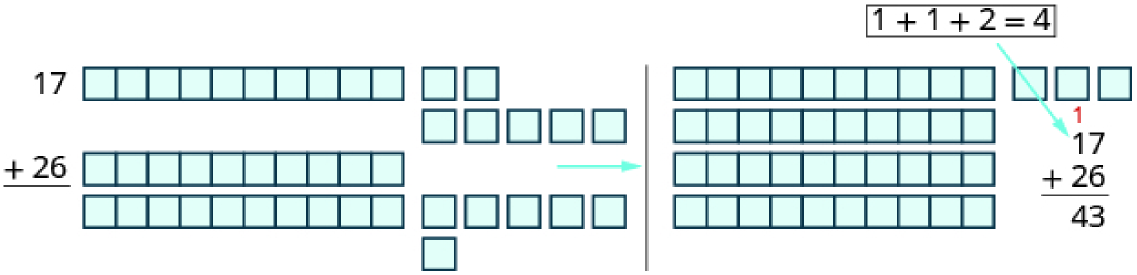
Note:
Exercise:

Problem: Add: $25 + 74$.

Solution:

$25 + 74 = 99$

In the previous example, the sum of the ones and the sum of the tens were both less than 10. But what happens if the sum is 10 or more? Let's use our base-10 model to find out. [link] shows the addition of 17 and 26 again.



When we add the ones, $7 + 6$, we get 13 ones. Because we have more than 10 ones, we can exchange 10 of the ones for 1 ten. Now we have 4 tens and 3 ones.

Without using the model, we show this as a small red 1 above the digits in the tens place.

When the sum in a place value column is greater than 9, we carry over to the next column to the left. Carrying is the same as regrouping by exchanging. For example, 10 ones for 1 ten or 10 tens for 1 hundred.

Note:
Add whole numbers.

Write the numbers so each place value lines up vertically.
Add the digits in each place value. Work from right to left starting with the ones place. If a sum in a place value is more than 9, carry to the next place value.
Continue adding each place value from right to left, adding each place value and carrying if needed.

Example:
Exercise:

Problem: Add: $43 + 69$.

Solution:
Solution

Write the numbers so the digits line up vertically.	$\begin{array}{r} 43 \\ \underline{+69} \end{array}$
Add the digits in each place. Add the ones: $3 + 9 = 12$	

Write the 2 in the ones place in the sum. Add the 1 ten to the tens place.	$\begin{array}{r} \overset{1}{4}3 \\ +69 \\ \hline 2 \end{array}$
Now add the tens: $1+4+6=11$ Write the 11 in the sum.	$\begin{array}{r} \overset{1}{4}3 \\ +69 \\ \hline 112 \end{array}$

Note:
Exercise:

Problem: Add: $35+98$.

Solution:

$35+98=133$

Note:
Exercise:

Problem: Add: $72+89$.

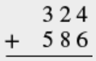
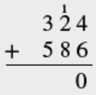
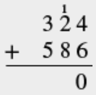
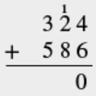
Solution:

$72+89=161$

Example:
Exercise:

Problem: Add: $324+586$.

Solution:
Solution

Write the numbers so the digits line up vertically.	
Add the digits in each place value. Add the ones: $4 + 6 = 10$ Write the 0 in the ones place in the sum and carry the 1 ten to the tens place.	
Add the tens: $1 + 2 + 8 = 11$ Write the 1 in the tens place in the sum and carry the 1 hundred to the hundreds	
Add the hundreds: $1 + 3 + 5 = 9$ Write the 9 in the hundreds place.	

Note:
Exercise:

Problem: Add: $456 + 376$.

Solution:

$456 + 376 = 832$

Note:
Exercise:

Problem: Add: $269 + 578$.

Solution:

$269 + 578 = 847$

Example:
Exercise:

Problem: Add: $1{,}683 + 479$.

Solution:
Solution

Write the numbers so the digits line up vertically.	$\begin{array}{r} 1{,}683 \\ +\,479 \\ \hline \end{array}$
Add the digits in each place value.	
Add the ones: $3 + 9 = 12$. Write the 2 in the ones place of the sum and carry the 1 ten to the tens place.	$\begin{array}{r} 1{,}6\overset{1}{8}3 \\ +\,479 \\ \hline 2 \end{array}$
Add the tens: $1 + 7 + 8 = 16$ Write the 6 in the tens place and carry the 1 hundred to the hundreds place.	$\begin{array}{r} 1{,}\overset{1}{6}\overset{1}{8}3 \\ +\,479 \\ \hline 62 \end{array}$

Add the hundreds: $1+6+4=11$ Write the 1 in the hundreds place and carry the 1 thousand to the thousands place.	$\begin{array}{r} 1{,}\overset{1}{6}\overset{1}{8}3 \\ +\,479 \\ \hline 162 \end{array}$
Add the thousands $1+1=2$. Write the 2 in the thousands place of the sum.	$\begin{array}{r} 1{,}\overset{1}{6}\overset{1}{8}3 \\ +\,479 \\ \hline 2{,}162 \end{array}$

When the addends have different numbers of digits, be careful to line up the corresponding place values starting with the ones and moving toward the left.

Note:
Exercise:

Problem: Add: $4{,}597+685$.

Solution:

$4{,}597+685=5{,}282$

Note:
Exercise:

Problem: Add: $5{,}837+695$.

Solution:

$5{,}837+695=6{,}532$

Example:

Exercise:

Problem: Add: $21{,}357 + 861 + 8{,}596$.

Solution:
Solution

Write the numbers so the place values line up vertically.	$\begin{array}{r} 21{,}357 \\ 861 \\ \underline{+\ 8{,}596} \end{array}$
Add the digits in each place value.	
Add the ones: $7 + 1 + 6 = 14$ Write the 4 in the ones place of the sum and carry the 1 to the tens place.	$\begin{array}{r} 21{,}3\overset{1}{5}7 \\ 861 \\ \underline{+\ 8{,}596} \\ 4 \end{array}$
Add the tens: $1 + 5 + 6 + 9 = 21$ Write the 1 in the tens place and carry the 2 to the hundreds place.	$\begin{array}{r} 21{,}\overset{2}{3}\overset{1}{5}7 \\ 861 \\ \underline{+\ 8{,}596} \\ 14 \end{array}$
Add the hundreds: $2 + 3 + 8 + 5 = 18$ Write the 8 in the hundreds place and carry the 1 to the thousands place.	$\begin{array}{r} 2\overset{1}{1}{,}\overset{2}{3}\overset{1}{5}7 \\ 861 \\ \underline{+\ 8{,}596} \\ 814 \end{array}$
Add the thousands $1 + 1 + 8 = 10$.	

Write the 0 in the thousands place and carry the 1 to the ten thousands place.	$\begin{array}{r} \overset{1}{2}\overset{1}{1},\overset{2}{3}\overset{1}{5}7 \\ 861 \\ +\ 8{,}596 \\ \hline 0814 \end{array}$
Add the ten-thousands $1 + 2 = 3$. Write the 3 in the ten thousands place in the sum.	$\begin{array}{r} \overset{1}{2}\overset{1}{1},\overset{2}{3}\overset{1}{5}7 \\ 861 \\ +\ 8{,}596 \\ \hline 30{,}814 \end{array}$

This example had three addends. We can add any number of addends using the same process as long as we are careful to line up the place values correctly.

Note:
Exercise:

Problem: Add: $46{,}195 + 397 + 6{,}281$.

Solution:

$46{,}195 + 397 + 6{,}281 = 52{,}873$

Note:
Exercise:

Problem: Add: $53{,}762 + 196 + 7{,}458$.

Solution:

$53{,}762 + 196 + 7{,}458 = 61{,}416$

Translate Word Phrases to Math Notation

Earlier in this section, we translated math notation into words. Now we’ll reverse the process. We’ll translate word phrases into math notation. Some of the word phrases that indicate addition are listed in [link].

Operation	**Words**	**Example**	**Expression**
Addition	plus sum increased by more than total of added to	1 plus 2 the sum of 3 and 4 5 increased by 6 8 more than 7 the total of 9 and 5 6 added to 4	$1+2$ $3+4$ $5+6$ $7+8$ $9+5$ $4+6$

Example:
Exercise:

Problem: Translate and simplify: the sum of 19 and 23.

Solution:
Solution

The word *sum* tells us to add. The words *of* 19 *and* 23 tell us the addends.

	The sum of 19 and 23
Translate.	$19+23$

Add.	42
	The sum of 19 and 23 is 42.

Note:
Exercise:

Problem: Translate and simplify: the sum of 17 and 26.

Solution:

Translate: 17 + 26; Simplify: 43

Note:
Exercise:

Problem: Translate and simplify: the sum of 28 and 14.

Solution:

Translate: 28 + 14; Simplify: 42

Example:
Exercise:

Problem: Translate and simplify: 28 increased by 31.

Solution:
Solution

The words *increased by* tell us to add. The numbers given are the addends.

	28 increased by 31.
Translate.	$28 + 31$
Add.	59
	So 28 increased by 31 is 59.

Note:
Exercise:

Problem: Translate and simplify: 29 increased by 76.

Solution:

Translate: $29 + 76$; Simplify 105

Note:
Exercise:

Problem: Translate and simplify: 37 increased by 69.

Solution:

Translate 37 + 69; Simplify 106

Add Whole Numbers in Applications

Now that we have practiced adding whole numbers, let’s use what we’ve learned to solve real-world problems. We’ll start by outlining a plan. First, we need to read the problem to determine what we are looking for. Then we write a word phrase that gives the information to find it. Next we translate the word phrase into math notation and then simplify. Finally, we write a sentence to answer the question.

Example:
Exercise:

Problem:

Hao earned grades of $87, 93, 68, 95,$ and 89 on the five tests of the semester. What is the total number of points he earned on the five tests?

Solution:
Solution

We are asked to find the total number of points on the tests.

Write a phrase.	the sum of points on the tests
Translate to math notation.	$87 + 93 + 68 + 95 + 89$
Then we simplify by adding.	
Since there are several numbers, we will write them vertically.	$\begin{array}{r} \overset{3}{8}7 \\ 93 \\ 68 \\ 95 \\ \underline{+89} \\ 432 \end{array}$
Write a sentence to answer the question.	Hao earned a total of 432 points.

Notice that we added *points*, so the sum is 432 *points*. It is important to include the appropriate units in all answers to applications problems.

Note:
Exercise:

Problem:

Mark is training for a bicycle race. Last week he rode 18 miles on Monday, 15 miles on Wednesday, 26 miles on Friday, 49 miles on Saturday, and 32 miles on Sunday. What is the total number of miles he rode last week?

Solution:

He rode 140 miles.

Note:
Exercise:

Problem:

Lincoln Middle School has three grades. The number of students in each grade is $230, 165,$ and 325. What is the total number of students?

Solution:

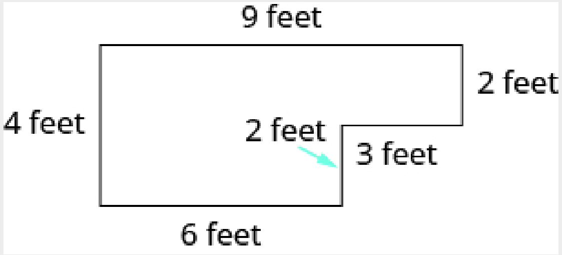
The total number is 720 students.

Some application problems involve shapes. For example, a person might need to know the distance around a garden to put up a fence or around a picture to frame it. The perimeter is the distance around a geometric figure. The perimeter of a figure is the sum of the lengths of its sides.

Example:

Exercise:

Problem: Find the perimeter of the patio shown.

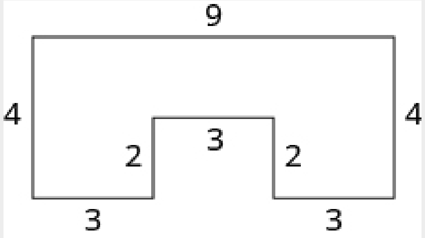


Solution:
Solution

We are asked to find the perimeter.	
Write a phrase.	the sum of the sides
Translate to math notation.	$4+6+2+3+2+9$
Simplify by adding.	26
Write a sentence to answer the question.	
We added feet, so the sum is 26 feet.	The perimeter of the patio is 26 feet.

Note:
Exercise:

Problem: Find the perimeter of each figure. All lengths are in inches.

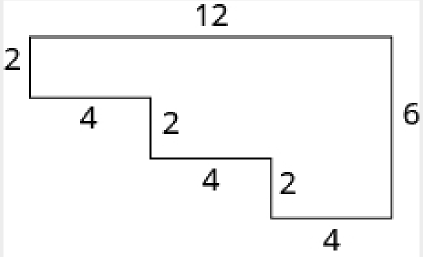


Solution:

The perimeter is 30 inches.

Note:
Exercise:

Problem: Find the perimeter of each figure. All lengths are in inches.



Solution:

The perimeter is 36 inches.

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Adding Two-Digit Numbers with base-10 blocks
- Adding Three-Digit Numbers with base-10 blocks

- Adding Whole Numbers

Key Concepts

- **Addition Notation** To describe addition, we can use symbols and words.

Operation	Notation	Expression	Read as	Result
Addition	$+$	$3+4$	three plus four	the sum of 3 and 4

- **Identity Property of Addition**
  - The sum of any number a and 0 is the number. $a+0=a$ $0+a=a$
- **Commutative Property of Addition**
  - Changing the order of the addends a and b does not change their sum. $a+b=b+a$.
- **Add whole numbers.**

  Write the numbers so each place value lines up vertically.
  Add the digits in each place value. Work from right to left starting with the ones place. If a sum in a place value is more than 9, carry to the next place value.
  Continue adding each place value from right to left, adding each place value and carrying if needed.

Practice Makes Perfect

Use Addition Notation

In the following exercises, translate the following from math expressions to words.

Exercise:

Problem: $5 + 2$

Solution:

five plus two; the sum of 5 and 2.

Exercise:

Problem: $6 + 3$

Exercise:

Problem: $13 + 18$

Solution:

thirteen plus eighteen; the sum of 13 and 18.

Exercise:

Problem: $15 + 16$

Exercise:

Problem: $214 + 642$

Solution:

two hundred fourteen plus six hundred forty-two; the sum of 214 and 642

Exercise:

Problem: $438 + 113$

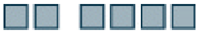
Model Addition of Whole Numbers

In the following exercises, model the addition.

Exercise:

Problem: $2 + 4$

Solution:



$2 + 4 = 6$

Exercise:

Problem: $5 + 3$

Exercise:

Problem: $8 + 4$

Solution:

$8 + 4 = 12$

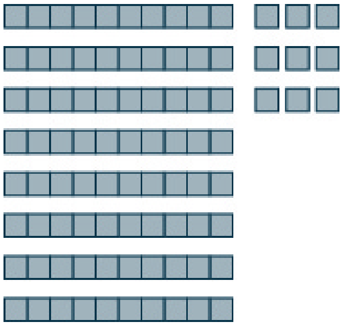
Exercise:

Problem: $5 + 9$

Exercise:

Problem: $14 + 75$

Solution:



$14 + 75 = 89$

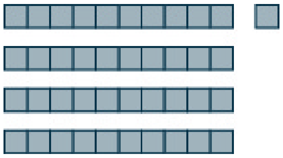
Exercise:

Problem: $15 + 63$

Exercise:

Problem: $16 + 25$

Solution:



$16 + 25 = 41$

Exercise:

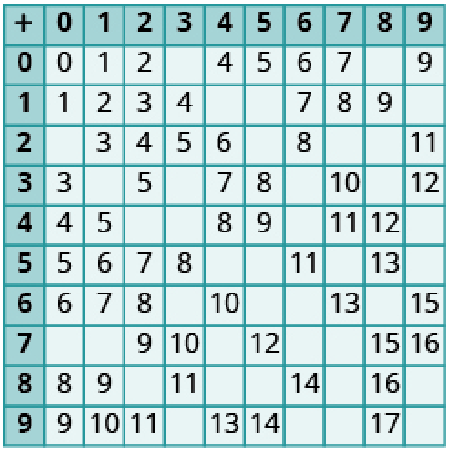
Problem: $14 + 27$

Add Whole Numbers

In the following exercises, fill in the missing values in each chart.

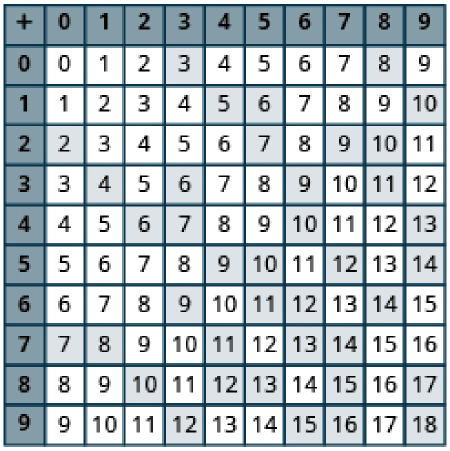
Exercise:

Problem:



+	0	1	2	3	4	5	6	7	8	9
0	0	1	2		4	5	6	7		9
1	1	2	3	4			7	8	9	
2		3	4	5	6		8			11
3	3		5		7	8		10		12
4	4	5			8	9		11	12	
5	5	6	7	8			11		13	
6	6	7	8		10			13		15
7			9	10		12			15	16
8	8	9		11			14		16	
9	9	10	11		13	14			17	

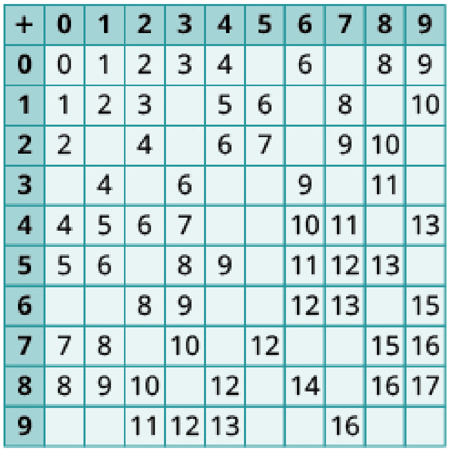
Solution:



+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Exercise:

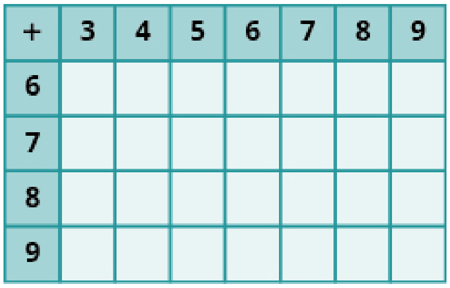
Problem:



+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4		6		8	9
1	1	2	3		5	6		8		10
2	2		4		6	7		9	10	
3		4		6			9		11	
4	4	5	6	7			10	11		13
5	5	6		8	9		11	12	13	
6			8	9			12	13		15
7	7	8		10		12			15	16
8	8	9	10		12		14		16	17
9			11	12	13			16		

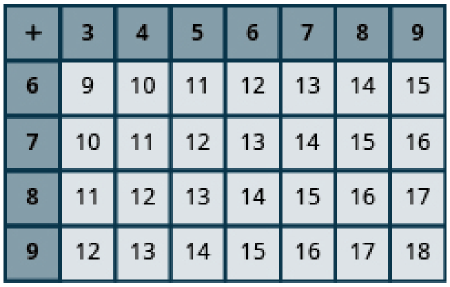
Exercise:

Problem:



+	3	4	5	6	7	8	9
6							
7							
8							
9							

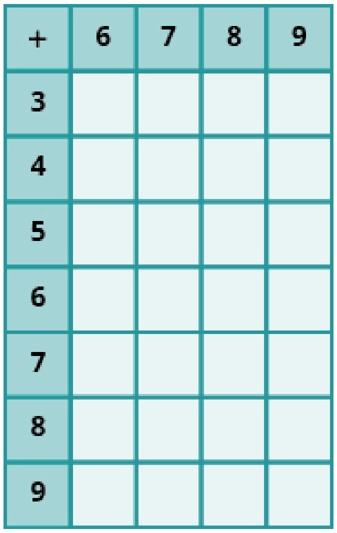
Solution:



+	3	4	5	6	7	8	9
6	9	10	11	12	13	14	15
7	10	11	12	13	14	15	16
8	11	12	13	14	15	16	17
9	12	13	14	15	16	17	18

Exercise:

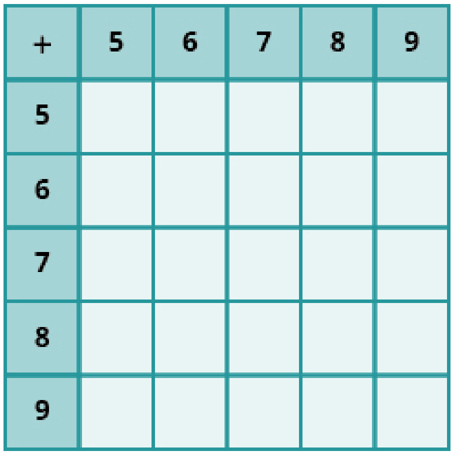
Problem:



+	6	7	8	9
3				
4				
5				
6				
7				
8				
9				

Exercise:

Problem:



+	5	6	7	8	9
5					
6					
7					
8					
9					

Solution:

+	5	6	7	8	9
5	10	11	12	13	14
6	11	12	13	14	15
7	12	13	14	15	16
8	13	14	15	16	17
9	14	15	16	17	18

Exercise:

Problem:

+	6	7	8	9
6				
7				
8				
9				

In the following exercises, add.

Exercise:

Problem:

ⓐ $0 + 13$
ⓑ $13 + 0$

Solution:

ⓐ 13
ⓑ 13

Exercise:

Problem:

ⓐ $0 + 5{,}280$
ⓑ $5{,}280 + 0$

Exercise:

Problem:

ⓐ $8 + 3$
ⓑ $3 + 8$

Solution:

ⓐ 11
ⓑ 11

Exercise:

Problem:

ⓐ $7 + 5$
ⓑ $5 + 7$

Exercise:

Problem: $45 + 33$

Solution:

78

Exercise:

Problem: $37 + 22$

Exercise:

Problem: $71 + 28$

Solution:

99

Exercise:

Problem: $43 + 53$

Exercise:

Problem: $26 + 59$

Solution:

85

Exercise:

Problem: $38 + 17$

Exercise:

Problem: $64 + 78$

Solution:

142

Exercise:

Problem: $92 + 39$

Exercise:

Problem: $168 + 325$

Solution:

493

Exercise:

Problem: $247 + 149$

Exercise:

Problem: $584 + 277$

Solution:

861

Exercise:

Problem: $175 + 648$

Exercise:

Problem: $832 + 199$

Solution:

1,031

Exercise:

Problem: $775 + 369$

Exercise:

Problem: $6{,}358 + 492$

Solution:

6,850

Exercise:

Problem: $9{,}184 + 578$

Exercise:

Problem: $3{,}740 + 18{,}593$

Solution:

22,333

Exercise:

Problem: $6{,}118 + 15{,}990$

Exercise:

Problem: $485{,}012 + 619{,}848$

Solution:

$1{,}104{,}860$

Exercise:

Problem: $368{,}911 + 857{,}289$

Exercise:

Problem: $24{,}731 + 592 + 3{,}868$

Solution:

$29{,}191$

Exercise:

Problem: $28{,}925 + 817 + 4{,}593$

Exercise:

Problem: $8{,}015 + 76{,}946 + 16{,}570$

Solution:

$101{,}531$

Exercise:

Problem: $6{,}291 + 54{,}107 + 28{,}635$

Translate Word Phrases to Math Notation

In the following exercises, translate each phrase into math notation and then simplify.

Exercise:

Problem: the sum of 13 and 18

Solution:

$13+18=31$

Exercise:

Problem: the sum of 12 and 19

Exercise:

Problem: the sum of 90 and 65

Solution:

$90+65=155$

Exercise:

Problem: the sum of 70 and 38

Exercise:

Problem: 33 increased by 49

Solution:

$33+49=82$

Exercise:

Problem: 68 increased by 25

Exercise:

Problem: 250 more than 599

Solution:

$250 + 599 = 849$

Exercise:

Problem: 115 more than 286

Exercise:

Problem: the total of 628 and 77

Solution:

$628 + 77 = 705$

Exercise:

Problem: the total of 593 and 79

Exercise:

Problem: $1{,}482$ added to 915

Solution:

$915 + 1{,}482 = 2{,}397$

Exercise:

Problem: $2{,}719$ added to 682

Add Whole Numbers in Applications

In the following exercises, solve the problem.

Exercise:

Problem:

Home remodeling Sophia remodeled her kitchen and bought a new range, microwave, and dishwasher. The range cost $\$1,100$, the microwave cost $\$250$, and the dishwasher cost $\$525$. What was the total cost of these three appliances?

Solution:

The total cost was $1,875.

Exercise:

Problem:

Sports equipment Aiden bought a baseball bat, helmet, and glove. The bat cost $\$299$, the helmet cost $\$35$, and the glove cost $\$68$. What was the total cost of Aiden’s sports equipment?

Exercise:

Problem:

Bike riding Ethan rode his bike 14 miles on Monday, 19 miles on Tuesday, 12 miles on Wednesday, 25 miles on Friday, and 68 miles on Saturday. What was the total number of miles Ethan rode?

Solution:

Ethan rode 138 miles.

Exercise:

Problem:

Business Chloe has a flower shop. Last week she made 19 floral arrangements on Monday, 12 on Tuesday, 23 on Wednesday, 29 on Thursday, and 44 on Friday. What was the total number of floral arrangements Chloe made?

Exercise:

Problem:

Apartment size Jackson lives in a 7 room apartment. The number of square feet in each room is $238, 120, 156, 196, 100, 132$, and 225. What is the total number of square feet in all 7 rooms?

Solution:

The total square footage in the rooms is 1,167 square feet.

Exercise:

Problem:

Weight Seven men rented a fishing boat. The weights of the men were $175, 192, 148, 169, 205, 181,$ and 225 pounds. What was the total weight of the seven men?

Exercise:

Problem:

Salary Last year Natalie’s salary was $\$82{,}572$. Two years ago, her salary was $\$79{,}316$, and three years ago it was $\$75{,}298$. What is the total amount of Natalie’s salary for the past three years?

Solution:

Natalie’s total salary is $237,186.

Exercise:

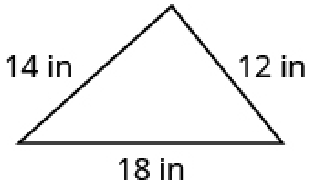
Problem:

Home sales Emma is a realtor. Last month, she sold three houses. The selling prices of the houses were $\$292{,}540$, $\$505{,}875$, and $\$423{,}699$. What was the total of the three selling prices?

In the following exercises, find the perimeter of each figure.

Exercise:

Problem:

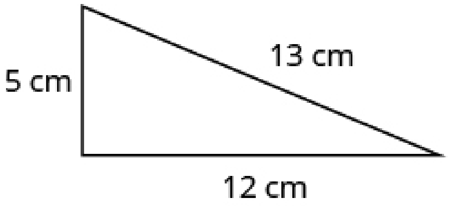


Solution:

The perimeter of the figure is 44 inches.

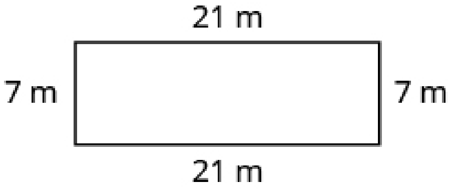
Exercise:

Problem:



Exercise:

Problem:

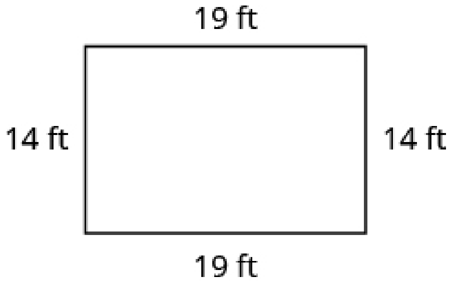


Solution:

The perimeter of the figure is 56 meters.

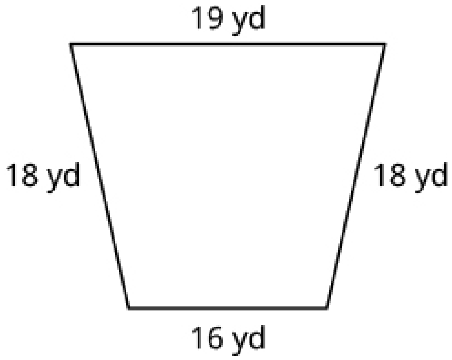
Exercise:

Problem:



Exercise:

Problem:

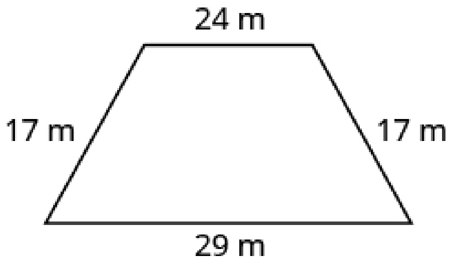


Solution:

The perimeter of the figure is 71 yards.

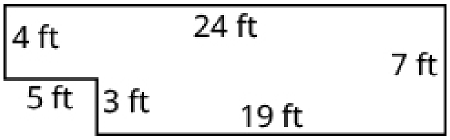
Exercise:

Problem:



Exercise:

Problem:

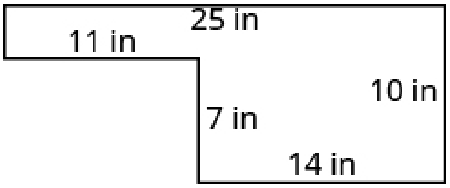


Solution:

The perimeter of the figure is 62 feet.

Exercise:

Problem:



Everyday Math

Exercise:

Problem:

Calories Paulette had a grilled chicken salad, ranch dressing, and a 16-ounce drink for lunch. On the restaurant’s nutrition chart, she saw that each item had the following number of calories:

Grilled chicken salad – 320 calories
Ranch dressing – 170 calories
16-ounce drink – 150 calories

What was the total number of calories of Paulette’s lunch?

Solution:

The total number of calories was 640.

Exercise:

Problem:

Calories Fred had a grilled chicken sandwich, a small order of fries, and a 12-oz chocolate shake for dinner. The restaurant’s nutrition chart lists the following calories for each item:

Grilled chicken sandwich – 420 calories
Small fries – 230 calories
12-oz chocolate shake – 580 calories

What was the total number of calories of Fred’s dinner?

Exercise:

Problem:

Test scores A students needs a total of 400 points on five tests to pass a course. The student scored 82, 91, 75, 88, and 70. Did the student pass the course?

Solution:

Yes, he scored 406 points.

Exercise:

Problem:

Elevators The maximum weight capacity of an elevator is 1150 pounds. Six men are in the elevator. Their weights are 210, 145, 183, 230, 159, and 164 pounds. Is the total weight below the elevators’ maximum capacity?

Writing Exercises

Exercise:

Problem:

How confident do you feel about your knowledge of the addition facts? If you are not fully confident, what will you do to improve your skills?

Solution:

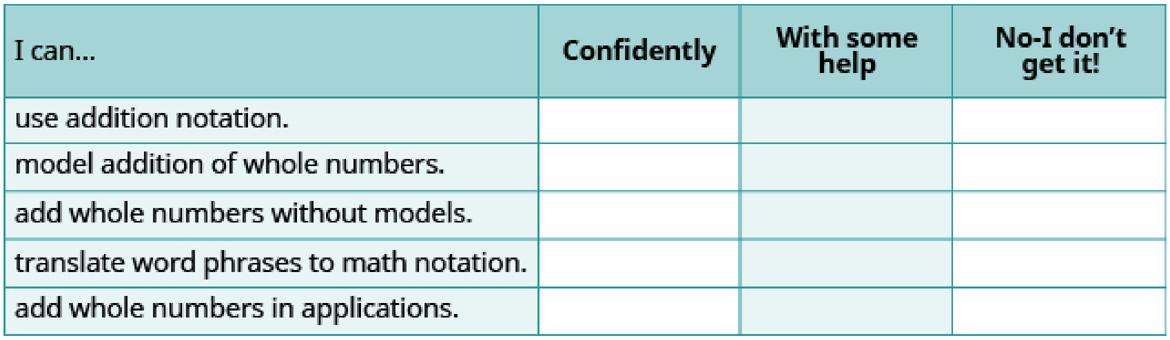
Answers will vary.

Exercise:

Problem: How have you used models to help you learn the addition facts?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
use addition notation.			
model addition of whole numbers.			
add whole numbers without models.			
translate word phrases to math notation.			
add whole numbers in applications.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Glossary

sum

The sum is the result of adding two or more numbers.

Subtract Whole Numbers

By the end of this section, you will be able to:

- Use subtraction notation
- Model subtraction of whole numbers
- Subtract whole numbers
- Translate word phrases to math notation
- Subtract whole numbers in applications

Note:
Before you get started, take this readiness quiz.

1. Model $3+4$ using base-ten blocks.
   If you missed this problem, review [link].
2. Add: $324+586$.
   If you missed this problem, review [link].

Use Subtraction Notation

Suppose there are seven bananas in a bowl. Elana uses three of them to make a smoothie. How many bananas are left in the bowl? To answer the question, we subtract three from seven. When we subtract, we take one number away from another to find the **difference**. The notation we use to subtract 3 from 7 is

Equation:

$$7-3$$

We read $7-3$ as *seven minus three* and the result is *the difference of seven and three*.

Note:
Subtraction Notation
To describe subtraction, we can use symbols and words.

Operation	**Notation**	**Expression**	**Read as**	**Result**
Subtraction	$-$	$7-3$	seven minus three	the difference of 7 and 3

Example:
Exercise:

Problem:

Translate from math notation to words: ⓐ $8-1$ ⓑ $26-14$.

Solution:
Solution

- ⓐ We read this as *eight minus one*. The result is *the difference of eight and one*.
- ⓑ We read this as *twenty-six minus fourteen*. The resuilt is *the difference of twenty-six and fourteen*.

Note:
Translate from math notation to words:

Exercise:

Problem:

ⓐ $12 - 4$
ⓑ $29 - 11$

Solution:

ⓐ twelve minus four; the difference of twelve and four
ⓑ twenty-nine minus eleven; the difference of twenty-nine and eleven

Note:
Translate from math notation to words:

Exercise:

Problem:

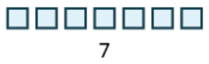
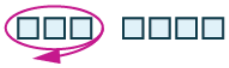
ⓐ $11 - 2$
ⓑ $29 - 12$

Solution:

ⓐ eleven minus two; the difference of eleven and two
ⓑ twenty-nine minus twelve; the difference of twenty-nine and twelve

Model Subtraction of Whole Numbers

A model can help us visualize the process of subtraction much as it did with addition. Again, we will use base-10 blocks. Remember a block represents 1 and a rod represents 10. Let’s start by modeling the subtraction expression we just considered, $7 - 3$.

We start by modeling the first number, 7.	
Now take away the second number, 3. We'll circle 3 blocks to show that we are taking them away.	
Count the number of blocks remaining.	
There are 4 ones blocks left.	We have shown that $7 - 3 = 4$.

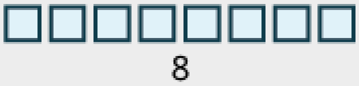
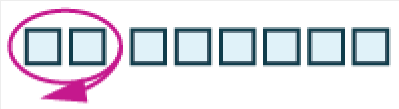
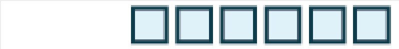
Note: Doing the Manipulative Mathematics activity Model Subtraction of Whole Numbers will help you develop a better understanding of subtracting whole numbers.

Example:

Exercise:

Problem: Model the subtraction: $8 - 2$.

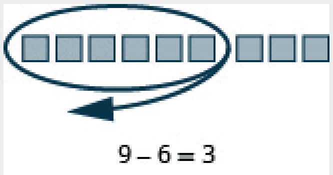
Solution:
Solution

$8 - 2$ means the difference of 8 and 2.	
Model the first, 8.	
Take away the second number, 2.	
Count the number of blocks remaining.	
There are 6 ones blocks left.	We have shown that $8 - 2 = 6$.

Note:
Exercise:

Problem: Model: 9 − 6.

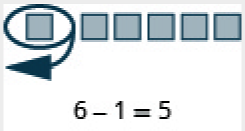
Solution:



Note:
Exercise:

Problem: Model: 6 − 1.

Solution:

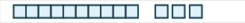
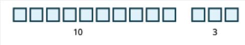
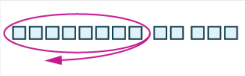
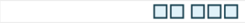


Example:
Exercise:

Problem: Model the subtraction: 13 − 8.

Solution:

Solution

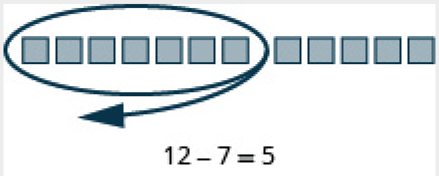
Model the first number, 13. We use 1 ten and 3 ones.	
Take away the second number, 8. However, there are not 8 ones, so we will exchange the 1 ten for 10 ones.	
Now we can take away 8 ones.	
Count the blocks remaining.	
There are five ones left.	We have shown that $13 - 8 = 5$.

As we did with addition, we can describe the models as ones blocks and tens rods, or we can simply say ones and tens.

Note:
Exercise:

Problem: Model the subtraction: $12 - 7$.

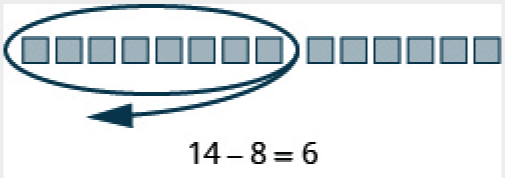
Solution:



Note:
Exercise:

Problem: Model the subtraction: 14 − 8.

Solution:

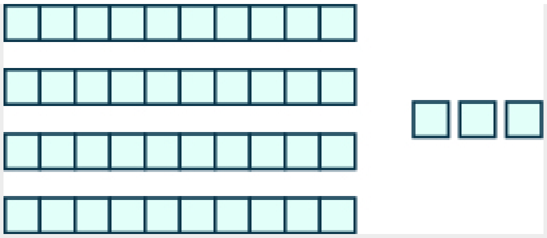


Example:
Exercise:

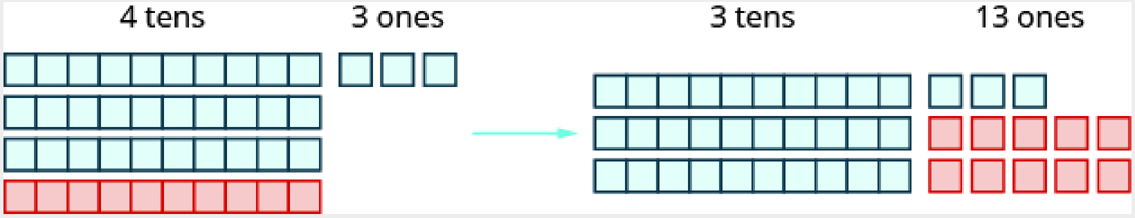
Problem: Model the subtraction: 43 − 26.

Solution:
Solution

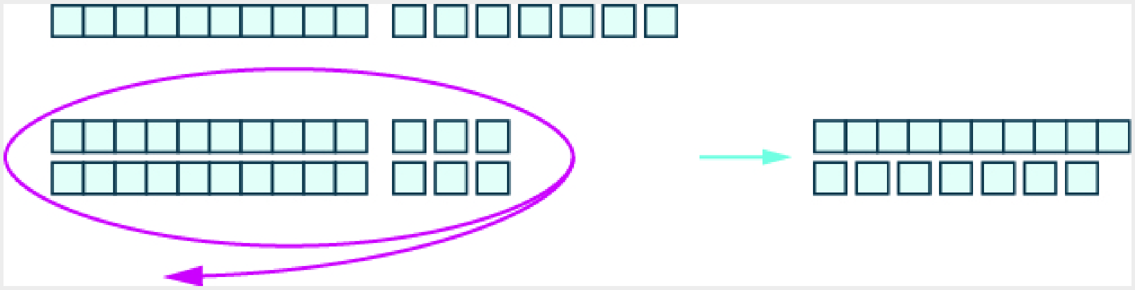
Because 43 − 26 means 43 take away 26, we begin by modeling the 43.



Now, we need to take away 26, which is 2 tens and 6 ones. We cannot take away 6 ones from 3 ones. So, we exchange 1 ten for 10 ones.



Now we can take away 2 tens and 6 ones.



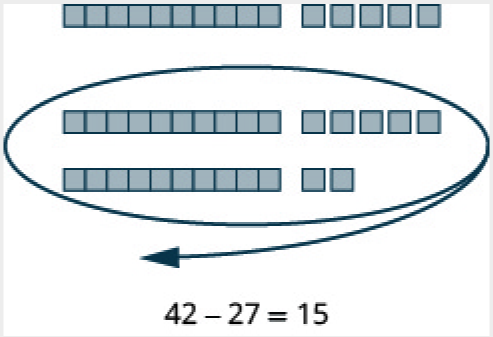
Count the number of blocks remaining. There is 1 ten and 7 ones, which is 17.

$43 - 26 = 17$

Note:
Exercise:

Problem: Model the subtraction: $42 - 27$.

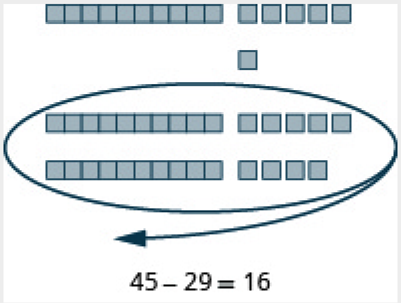
Solution:



Note:
Exercise:

Problem: Model the subtraction: $45 - 29$.

Solution:



Subtract Whole Numbers

Addition and subtraction are inverse operations. Addition undoes subtraction, and subtraction undoes addition.

We know $7-3=4$ because $4+3=7$. Knowing all the addition number facts will help with subtraction. Then we can check subtraction by adding. In the examples above, our subtractions can be checked by addition.

Equation:

$$\begin{array}{ccc} 7-3=4 & \text{because} & 4+3=7 \\ 13-8=5 & \text{because} & 5+8=13 \\ 43-26=17 & \text{because} & 17+26=43 \end{array}$$

Example:
Exercise:

Problem: Subtract and then check by adding:

ⓐ $9-7$
ⓑ $8-3$.

Solution:
Solution

ⓐ	
	$9-7$
Subtract 7 from 9.	2
Check with addition. $2+7=9$✓	

ⓑ	
	$8 - 3$
Subtract 3 from 8.	5
Check with addition. $5 + 3 = 8$✓	

Note:
Exercise:

Problem: Subtract and then check by adding:

$7 - 0$

Solution:

7 − 0 = 7; 7 + 0 = 7

Note:
Exercise:

Problem: Subtract and then check by adding:

$6 - 2$

Solution:

6 − 2 = 4; 2 + 4 = 6

To subtract numbers with more than one digit, it is usually easier to write the numbers vertically in columns just as we did for addition. Align the digits by place value, and then subtract each column starting with the ones and then working to the left.

Example:
Exercise:

Problem: Subtract and then check by adding: $89 - 61$.

Solution:
Solution

Write the numbers so the ones and tens digits line up vertically.	$\begin{array}{r} 89 \\ \underline{-61} \end{array}$
Subtract the digits in each place value. Subtract the ones: $9 - 1 = 8$ Subtract the tens: $8 - 6 = 2$	$\begin{array}{r} 89 \\ \underline{-61} \\ 28 \end{array}$
Check using addition. $\begin{array}{r} 28 \\ \underline{+61} \\ 89 \end{array}$	

Our answer is correct.

Note:
Exercise:

Problem: Subtract and then check by adding: $86 - 54$.

Solution:

86 − 54 = 32 because 54 + 32 = 86

Note:
Exercise:

Problem: Subtract and then check by adding: $99 - 74$.

Solution:

99 − 74 = 25 because 74 + 25 = 99

When we modeled subtracting 26 from 43, we exchanged 1 ten for 10 ones. When we do this without the model, we say we borrow 1 from the tens place and add 10 to the ones place.

Note:
Find the difference of whole numbers.

Write the numbers so each place value lines up vertically.
Subtract the digits in each place value. Work from right to left starting with the ones place. If the digit on top is less than the digit below, borrow as needed.
Continue subtracting each place value from right to left, borrowing if

needed.
Check by adding.

Example:
Exercise:

Problem: Subtract: $43 - 26$.

Solution:
Solution

Write the numbers so each place value lines up vertically.	
Subtract the ones. We cannot subtract 6 from 3, so we borrow 1 ten. This makes 3 tens and 13 ones. We write these numbers above each place and cross out the original digits.	
Now we can subtract the ones. $13 - 6 = 7$. We write the 7 in the ones place in the difference.	
Now we subtract the tens. $3 - 2 = 1$. We write the 1 in the tens place in the difference.	
Check by adding.	

$$\begin{array}{r} 17 \\ +26 \\ \hline 43\checkmark \end{array}$$

Our answer is correct.

Note:
Exercise:

Problem: Subtract and then check by adding: $93 - 58$.

Solution:

$93 - 58 = 35$ because $58 + 35 = 93$

Note:
Exercise:

Problem: Subtract and then check by adding: $81 - 39$.

Solution:

$81 - 39 = 42$ because $42 + 39 = 81$

Example:
Exercise:

Problem: Subtract and then check by adding: $207 - 64$.

Solution:
Solution

Write the numbers so each place value lines up vertically.	
Subtract the ones. $7 - 4 = 3$. Write the 3 in the ones place in the difference. Write the 3 in the ones place in the difference.	
Subtract the tens. We cannot subtract 6 from 0 so we borrow 1 hundred and add 10 tens to the 0 tens we had. This makes a total of 10 tens. We write 10 above the tens place and cross out the 0. Then we cross out the 2 in the hundreds place and write 1 above it.	
Now we subtract the tens. $10 - 6 = 4$. We write the 4 in the tens place in the difference.	
Finally, subtract the hundreds. There is no digit in the hundreds place in the bottom number so we can imagine a 0 in that place. Since $1 - 0 = 1$, we write 1 in the hundreds place in the difference.	
Check by adding. $\begin{array}{r} \overset{1}{1}43 \\ +\ 64 \\ \hline 207\checkmark \end{array}$	

Our answer is correct.

Note:
Exercise:

Problem: Subtract and then check by adding: 439 − 52.

Solution:

439 − 52 = 387 because 387 + 52 = 439

Note:
Exercise:

Problem: Subtract and then check by adding: 318 − 75.

Solution:

318 − 75 = 243 because 243 + 75 = 318

Example:
Exercise:

Problem: Subtract and then check by adding: 910 − 586.

Solution:
Solution

Write the numbers so each place value lines up vertically.	
Subtract the ones. We cannot subtract 6 from 0, so we borrow 1 ten and add 10 ones to the 10 ones we had. This makes 10 ones. We write a 0 above the tens place and cross out the 1. We write the 10 above the ones place and cross out the 0. Now we can subtract the ones. $10 - 6 = 4$.	
Write the 4 in the ones place of the difference.	
Subtract the tens. We cannot subtract 8 from 0, so we borrow 1 hundred and add 10 tens to the 0 tens we had, which gives us 10 tens. Write 8 above the hundreds place and cross out the 9. Write 10 above the tens place.	
Now we can subtract the tens. $10 - 8 = 2$.	
Subtract the hundreds place. $8 - 5 = 3$ Write the 3 in the hundreds place in the difference.	
Check by adding. $\begin{array}{rr} & \overset{1}{3}\overset{1}{2}4 \\ + & 586 \\ \hline & 910 \ \checkmark \end{array}$ Our answer is correct.	

Note:
Exercise:

Problem: Subtract and then check by adding: $832 - 376$.

Solution:

832 − 376 = 456 because 456 + 376 = 832

Note:
Exercise:

Problem: Subtract and then check by adding: $847 - 578$.

Solution:

847 − 578 = 269 because 269 + 578 = 847

Example:
Exercise:

Problem: Subtract and then check by adding: $2{,}162 - 479$.

Solution:
Solution

Write the numbers so each place values line up	

vertically.	$\begin{array}{r} 2,162 \\ -\ 479 \\ \hline \end{array}$
Subtract the ones. Since we cannot subtract 9 from 2, borrow 1 ten and add 10 ones to the 2 ones to make 12 ones. Write 5 above the tens place and cross out the 6. Write 12 above the ones place and cross out the 2.	$\begin{array}{r} {\scriptstyle 5\ 12} \\ 2,1\cancel{6}\cancel{2} \\ -\ 479 \\ \hline \end{array}$
Now we can subtract the ones.	$12 - 9 = 3$
Write 3 in the ones place in the difference.	$\begin{array}{r} {\scriptstyle 5\ 12} \\ 2,1\cancel{6}\cancel{2} \\ -\ 479 \\ \hline 3 \end{array}$
Subtract the tens. Since we cannot subtract 7 from 5, borrow 1 hundred and add 10 tens to the 5 tens to make 15 tens. Write 0 above the hundreds place and cross out the 1. Write 15 above the tens place.	$\begin{array}{r} {\scriptstyle 15\ \ } \\ {\scriptstyle 0\ \cancel{5}\ 12} \\ 2,\cancel{1}\cancel{6}\cancel{2} \\ -\ 479 \\ \hline 3 \end{array}$
Now we can subtract the tens.	$15 - 7 = 8$
Write 8 in the tens place in the difference.	$\begin{array}{r} {\scriptstyle 0\ 15\ 12} \\ 2,\cancel{1}\cancel{6}\cancel{2} \\ -\ 479 \\ \hline 83 \end{array}$
Now we can subtract the hundreds.	

	$\begin{array}{rrrr} & 10 & & \\ 1 & \cancel{0} & 15 & 12 \\ \cancel{2}, & \cancel{1} & \cancel{6} & \cancel{2} \\ - & 4 & 7 & 9 \\ \hline & & 8 & 3 \end{array}$
Write 6 in the hundreds place in the difference.	$\begin{array}{rrrr} 1 & 10 & 15 & 12 \\ \cancel{2}, & \cancel{1} & \cancel{6} & \cancel{2} \\ - & 4 & 7 & 9 \\ \hline & 6 & 8 & 3 \end{array}$
Subtract the thousands. There is no digit in the thousands place of the bottom number, so we imagine a 0. $1 - 0 = 1$. Write 1 in the thousands place of the difference.	$\begin{array}{rrrr} 1 & 10 & \cancel{15} & \cancel{12} \\ \cancel{2}, & \cancel{1} & \cancel{6} & \cancel{2} \\ - & 4 & 7 & 9 \\ \hline 1, & 6 & 8 & 3 \end{array}$
Check by adding. $\begin{array}{r} \overset{1}{1},\overset{1}{6}\overset{1}{8}3 \\ +\ \ 479 \\ \hline 2,162\checkmark \end{array}$	

Our answer is correct.

Note:
Exercise:

Problem: Subtract and then check by adding: 4,585 − 697.

Solution:

4,585 − 697 = 3,888 because 3,888 + 697 = 4,585

Note:
Exercise:

Problem: Subtract and then check by adding: $5{,}637 - 899$.

Solution:

5,637 − 899 = 4,738 because 4,738 + 899 = 5,637

Translate Word Phrases to Math Notation

As with addition, word phrases can tell us to operate on two numbers using subtraction. To translate from a word phrase to math notation, we look for key words that indicate subtraction. Some of the words that indicate subtraction are listed in [link].

Operation	Word Phrase	Example	Expression
Subtraction	minus	5 minus 1	$5 - 1$
	difference	the difference of 9 and 4	$9 - 4$

Operation	Word Phrase	Example	Expression
	decreased by	7 decreased by 3	$7-3$
	less than	5 less than 8	$8-5$
	subtracted from	1 subtracted from 6	$6-1$

Example:
Exercise:

Problem: Translate and then simplify:

ⓐ the difference of 13 and 8
ⓑ subtract 24 from 43

Solution:
Solution

- ⓐ

  The word *difference* tells us to subtract the two numbers. The numbers stay in the same order as in the phrase.

	the difference of 13 and 8

Translate.	$13 - 8$
Simplify.	5

- ⓑ

  The words *subtract from* tells us to take the second number away from the first. We must be careful to get the order correct.

	subtract 24 from 43
Translate.	$43 - 24$
Simplify.	19

Note:
Exercise:

Problem: Translate and simplify:

ⓐ the difference of 14 and 9
ⓑ subtract 21 from 37

Solution:

ⓐ $14 - 9 = 5$
ⓑ $37 - 21 = 16$

Note:
Exercise:

Problem: Translate and simplify:

ⓐ 11 decreased by 6
ⓑ 18 less than 67

Solution:

ⓐ $11 - 6 = 5$
ⓑ $67 - 18 = 49$

Subtract Whole Numbers in Applications

To solve applications with subtraction, we will use the same plan that we used with addition. First, we need to determine what we are asked to find. Then we write a phrase that gives the information to find it. We translate the phrase into math notation and then simplify to get the answer. Finally, we write a sentence to answer the question, using the appropriate units.

Example:
Exercise:

Problem:

The temperature in Chicago one morning was 73 degrees Fahrenheit. A cold front arrived and by noon the temperature was 27 degrees Fahrenheit. What was the difference between the temperature in the morning and the temperature at noon?

Solution:
Solution

We are asked to find the difference between the morning temperature and the noon temperature.

Write a phrase.	the difference of 73 and 27
Translate to math notation. *Difference* tells us to subtract.	$73-27$
Then we do the subtraction.	$\begin{array}{r} \overset{6}{\cancel{7}}\overset{13}{\cancel{3}} \\ -\quad 27 \\ \hline 46 \end{array}$
Write a sentence to answer the question.	The difference in temperatures was 46 degrees Fahrenheit.

Note:
Exercise:

Problem:

The high temperature on June 1^{st} in Boston was 77 degrees Fahrenheit, and the low temperature was 58 degrees Fahrenheit. What was the difference between the high and low temperatures?

Solution:

The difference is 19 degrees Fahrenheit.

Note:
Exercise:

Problem:

The weather forecast for June 2 in St Louis predicts a high temperature of 90 degrees Fahrenheit and a low of 73 degrees Fahrenheit. What is the difference between the predicted high and low temperatures?

Solution:

The difference is 17 degrees Fahrenheit.

Example:
Exercise:

Problem:

A washing machine is on sale for \$399. Its regular price is \$588. What is the difference between the regular price and the sale price?

Solution:
Solution

We are asked to find the difference between the regular price and the sale price.

Write a phrase.	the difference between 588 and 399
Translate to math notation.	$588 - 399$
Subtract.	$\begin{array}{r} \overset{4}{\cancel{5}}\overset{17}{\cancel{8}}\overset{18}{\cancel{8}} \\ -\ 399 \\ \hline 189 \end{array}$
Write a sentence to answer the question.	The difference between the regular price and the sale price is \$189.

Note:
Exercise:

Problem:

A television set is on sale for \$499. Its regular price is \$648. What is the difference between the regular price and the sale price?

Solution:

The difference is $149.

Note:
Exercise:

Problem:

A patio set is on sale for $\$149$. Its regular price is $\$285$. What is the difference between the regular price and the sale price?

Solution:

The difference is $136.

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Model subtraction of two-digit whole numbers
- Model subtraction of three-digit whole numbers
- Subtract Whole Numbers

Key Concepts

Operation	**Notation**	**Expression**	**Read as**	**Result**
Subtraction	$-$	$7-3$	seven minus three	the difference of 7 and 3

- **Subtract whole numbers.**

  Write the numbers so each place value lines up vertically.
  Subtract the digits in each place value. Work from right to left starting with the ones place. If the digit on top is less than the digit below, borrow as needed.
  Continue subtracting each place value from right to left, borrowing if needed.
  Check by adding.

Practice Makes Perfect

Use Subtraction Notation

In the following exercises, translate from math notation to words.
Exercise:

Problem: $15-9$

Solution:

fifteen minus nine; the difference of fifteen and nine

Exercise:

Problem: $18-16$

Exercise:

Problem: $42 - 35$

Solution:

forty-two minus thirty-five; the difference of forty-two and thirty-five

Exercise:

Problem: $83 - 64$

Exercise:

Problem: $675 - 350$

Solution:

hundred seventy-five minus three hundred fifty; the difference of six hundred seventy-five and three hundred fifty

Exercise:

Problem: $790 - 525$

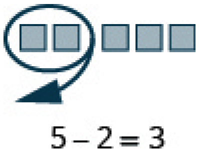
Model Subtraction of Whole Numbers

In the following exercises, model the subtraction.

Exercise:

Problem: $5 - 2$

Solution:



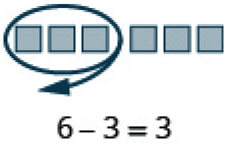
Exercise:

Problem: $8 - 4$

Exercise:

Problem: $6 - 3$

Solution:



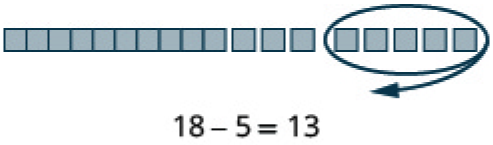
Exercise:

Problem: $7 - 5$

Exercise:

Problem: $18 - 5$

Solution:



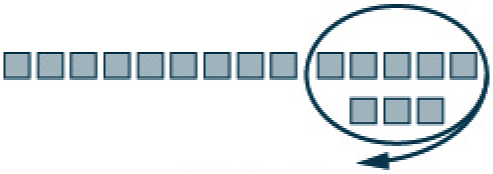
Exercise:

Problem: $19 - 8$

Exercise:

Problem: $17 - 8$

Solution:



$17 - 8 = 9$

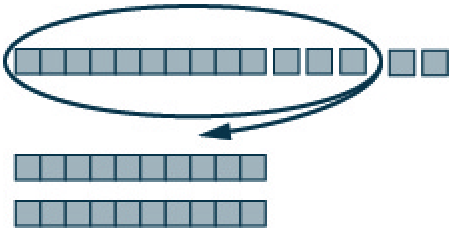
Exercise:

Problem: $17 - 9$

Exercise:

Problem: $35 - 13$

Solution:



$35 - 13 = 22$

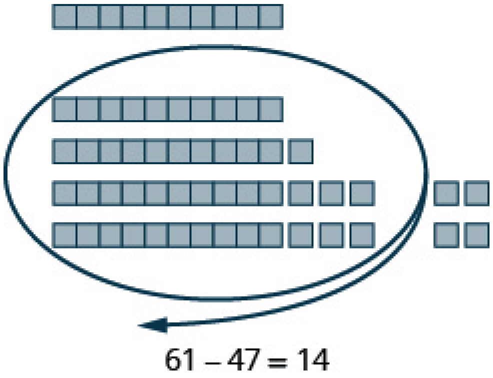
Exercise:

Problem: $32 - 11$

Exercise:

Problem: $61 - 47$

Solution:



Exercise:

Problem: $55 - 36$

Subtract Whole Numbers

In the following exercises, subtract and then check by adding.

Exercise:

Problem: $9 - 4$

Solution:

5

Exercise:

Problem: $9 - 3$

Exercise:

Problem: $8 - 0$

Solution:

8

Exercise:

Problem: $2 - 0$

Exercise:

Problem: $38 - 16$

Solution:

22

Exercise:

Problem: $45 - 21$

Exercise:

Problem: $85 - 52$

Solution:

33

Exercise:

Problem: $99 - 47$

Exercise:

Problem: $493 - 370$

Solution:

123

Exercise:

Problem: $268 - 106$

Exercise:

Problem: $5{,}946 - 4{,}625$

Solution:

$1{,}321$

Exercise:

Problem: $7{,}775 - 3{,}251$

Exercise:

Problem: $75 - 47$

Solution:

28

Exercise:

Problem: $63 - 59$

Exercise:

Problem: $461 - 239$

Solution:

222

Exercise:

Problem: $486 - 257$

Exercise:

Problem: $525 - 179$

Solution:

346

Exercise:

Problem: $542 - 288$

Exercise:

Problem: $6{,}318 - 2{,}799$

Solution:

$3{,}519$

Exercise:

Problem: $8{,}153 - 3{,}978$

Exercise:

Problem: $2{,}150 - 964$

Solution:

1,186

Exercise:

Problem: $4{,}245 - 899$

Exercise:

Problem: $43{,}650 - 8{,}982$

Solution:

34,668

Exercise:

Problem: $35{,}162 - 7{,}885$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate and simplify.

Exercise:

Problem: The difference of 10 and 3

Solution:

$10 - 3$; 7

Exercise:

Problem: The difference of 12 and 8

Exercise:

Problem: The difference of 15 and 4

Solution:

15 − 4; 11

Exercise:

Problem: The difference of 18 and 7

Exercise:

Problem: Subtract 6 from 9

Solution:

9 − 6; 3

Exercise:

Problem: Subtract 8 from 9

Exercise:

Problem: Subtract 28 from 75

Solution:

75 − 28; 47

Exercise:

Problem: Subtract 59 from 81

Exercise:

Problem: 45 decreased by 20

Solution:

45 − 20; 25

Exercise:

Problem: 37 decreased by 24

Exercise:

Problem: 92 decreased by 67

Solution:

92 − 67; 25

Exercise:

Problem: 75 decreased by 49

Exercise:

Problem: 12 less than 16

Solution:

16 − 12; 4

Exercise:

Problem: 15 less than 19

Exercise:

Problem: 38 less than 61

Solution:

61 − 38; 23

Exercise:

Problem: 47 less than 62

Mixed Practice

In the following exercises, simplify.

Exercise:

Problem: $76 - 47$

Solution:

29

Exercise:

Problem: $91 - 53$

Exercise:

Problem: $256 - 184$

Solution:

72

Exercise:

Problem: $305 - 262$

Exercise:

Problem: $719 + 341$

Solution:

1,060

Exercise:

Problem: $647 + 528$

Exercise:

Problem: $2{,}015 - 1{,}993$

Solution:

22

Exercise:

Problem: $2{,}020 - 1{,}984$

In the following exercises, translate and simplify.

Exercise:

Problem: Seventy-five more than thirty-five

Solution:

75 + 35; 110

Exercise:

Problem: Sixty more than ninety-three

Exercise:

Problem: 13 less than 41

Solution:

41 − 13; 28

Exercise:

Problem: 28 less than 36

Exercise:

Problem: The difference of 100 and 76

Solution:

100 − 76; 24

Exercise:

Problem: The difference of 1,000 and 945

Subtract Whole Numbers in Applications

In the following exercises, solve.

Exercise:

Problem:

Temperature The high temperature on June 2 in Las Vegas was 80 degrees and the low temperature was 63 degrees. What was the difference between the high and low temperatures?

Solution:

The difference between the high and low temperature was 17 degrees

Exercise:

Problem:

Temperature The high temperature on June 1 in Phoenix was 97 degrees and the low was 73 degrees. What was the difference between the high and low temperatures?

Exercise:

Problem:

Class size Olivia’s third grade class has 35 children. Last year, her second grade class had 22 children. What is the difference between the number of children in Olivia’s third grade class and her second grade class?

Solution:

The difference between the third grade and second grade was 13 children.

Exercise:

Problem:

Class size There are 82 students in the school band and 46 in the school orchestra. What is the difference between the number of students in the band and the orchestra?

Exercise:

Problem:

Shopping A mountain bike is on sale for $399. Its regular price is $650. What is the difference between the regular price and the sale price?

Solution:

The difference between the regular price and sale price is $251.

Exercise:

Problem:

Shopping A mattress set is on sale for $755. Its regular price is $1,600. What is the difference between the regular price and the sale price?

Exercise:

Problem:

Savings John wants to buy a laptop that costs $840. He has $685 in his savings account. How much more does he need to save in order to buy the laptop?

Solution:

John needs to save $155 more.

Exercise:

Problem:

Banking Mason had $1,125 in his checking account. He spent $892. How much money does he have left?

Everyday Math

Exercise:

Problem:

Road trip Noah was driving from Philadelphia to Cincinnati, a distance of 502 miles. He drove 115 miles, stopped for gas, and then drove another 230 miles before lunch. How many more miles did he have to travel?

Solution:

157 miles

Exercise:

Problem:

Test Scores Sara needs 350 points to pass her course. She scored $75, 50, 70,$ and 80 on her first four tests. How many more points does Sara need to pass the course?

Writing Exercises

Exercise:

Problem: Explain how subtraction and addition are related.

Solution:

Answers may vary.

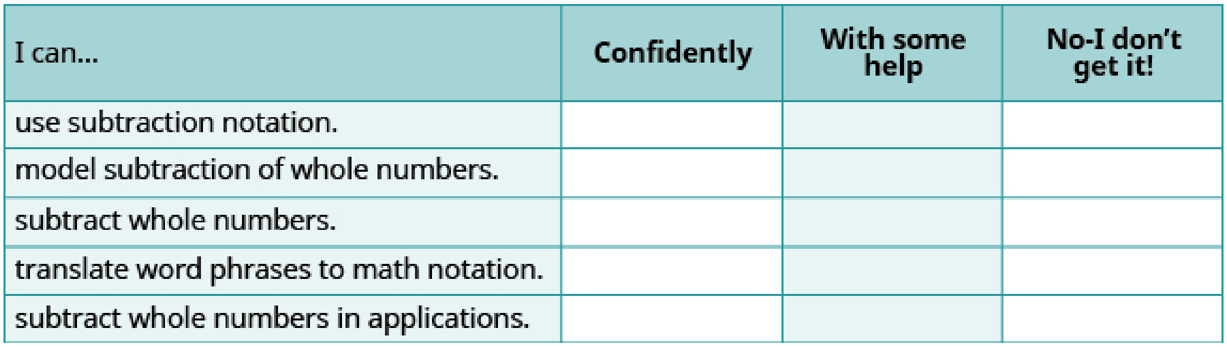
Exercise:

Problem:

How does knowing addition facts help you to subtract numbers?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
use subtraction notation.			
model subtraction of whole numbers.			
subtract whole numbers.			
translate word phrases to math notation.			
subtract whole numbers in applications.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Glossary

difference

The difference is the result of subtracting two or more numbers.

Multiply Whole Numbers

By the end of this section, you will be able to:

- Use multiplication notation
- Model multiplication of whole numbers
- Multiply whole numbers
- Translate word phrases to math notation
- Multiply whole numbers in applications

Note:
Before you get started, take this readiness quiz.

1. Add: $1,683+479$.
   If you missed this problem, review [link].
2. Subtract: $605-321$.
   If you missed this problem, review [link].

Use Multiplication Notation

Suppose you were asked to count all these pennies shown in [link].

Would you count the pennies individually? Or would you count the number of pennies in each row and add that number 3 times.
Equation:

$$8+8+8$$

Multiplication is a way to represent repeated addition. So instead of adding 8 three times, we could write a multiplication expression.
Equation:

$$3\times 8$$

We call each number being multiplied a factor and the result the **product**. We read 3×8 as *three times eight*, and the result as *the product of three and eight*.

There are several symbols that represent multiplication. These include the symbol $\times$ as well as the dot, $\cdot$, and parentheses $(\,)$.

Note:
Operation Symbols for Multiplication
To describe multiplication, we can use symbols and words.

Operation	**Notation**	**Expression**	**Read as**	**Result**
Multiplication	$\times$ $\cdot$ $(\ )$	3×8 $3 \cdot 8$ $3(8)$	three times eight	the product of 3 and 8

Example:
Exercise:

Problem: Translate from math notation to words:

ⓐ 7×6
ⓑ $12 \cdot 14$
ⓒ $6(13)$

Solution:
Solution

- ⓐ We read this as *seven times six* and the result is *the product of seven and six*.
- ⓑ We read this as *twelve times fourteen* and the result is *the product of twelve and fourteen*.
- ⓒ We read this as *six times thirteen* and the result is *the product of six and thirteen*.

Note:
Exercise:

Problem: Translate from math notation to words:

ⓐ 8×7
ⓑ $18 \cdot 11$

Solution:

ⓐ eight times seven ; the product of eight and seven
ⓑ eighteen times eleven ; the product of eighteen and eleven

Note:
Exercise:

Problem: Translate from math notation to words:

ⓐ $(13)(7)$
ⓑ $5(16)$

Solution:

ⓐ thirteen times seven ; the product of thirteen and seven
ⓑ five times sixteen; the product of five and sixteen

Model Multiplication of Whole Numbers

There are many ways to model multiplication. Unlike in the previous sections where we used base-10 blocks, here we will use counters to help us understand the meaning of multiplication. A counter is any object that can be used for counting. We will use round blue counters.

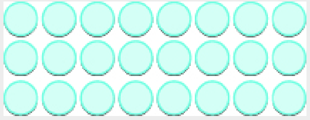
Example:
Exercise:

Problem: Model: 3×8.

Solution:
Solution

To model the product 3×8, we'll start with a row of 8 counters.

The other factor is 3, so we'll make 3 rows of 8 counters.



Now we can count the result. There are 24 counters in all.

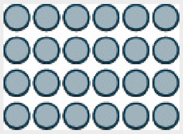
$3 \times 8 = 24$

If you look at the counters sideways, you'll see that we could have also made 8 rows of 3 counters. The product would have been the same. We'll get back to this idea later.

Note:
Exercise:

Problem: Model each multiplication: 4×6.

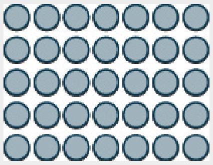
Solution:



Note:
Exercise:

Problem: Model each multiplication: 5×7.

Solution:



Multiply Whole Numbers

In order to multiply without using models, you need to know all the one digit multiplication facts. Make sure you know them fluently before proceeding in this section.

[link] shows the multiplication facts. Each box shows the product of the number down the left column and the number across the top row. If you are unsure about a product, model it. It is important that you memorize any number facts you do not already know so you will be ready to multiply larger numbers.

×	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0	0	0	0	0	0	0	0	0	0	0

×	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

What happens when you multiply a number by zero? You can see that the product of any number and zero is zero. This is called the Multiplication Property of Zero.

Note:
Multiplication Property of Zero
The product of any number and 0 is 0.
Equation:

$$a \cdot 0 = 0$$
$$0 \cdot a = 0$$

Example:
Exercise:

Problem: Multiply:

ⓐ $0 \cdot 11$
ⓑ $(42)0$

Solution:
Solution

ⓐ	$0 \cdot 11$
The product of any number and zero is zero.	0
ⓑ	$(42)0$
Multiplying by zero results in zero.	0

Note:
Exercise:

Problem: Find each product:

ⓐ $0 \cdot 19$
ⓑ $(39)0$

Solution:

ⓐ 0
ⓑ 0

Note:
Exercise:

Problem: Find each product:

ⓐ $0 \cdot 24$
ⓑ $(57)0$

Solution:

ⓐ 0
ⓑ 0

What happens when you multiply a number by one? Multiplying a number by one does not change its value. We call this fact the Identity Property of Multiplication, and 1 is called the multiplicative identity.

Note:

Identity Property of Multiplication
The product of any number and 1 is the number.
Equation:

$$1 \cdot a = a$$
$$a \cdot 1 = a$$

Example:
Exercise:

Problem: Multiply:

ⓐ (11)1
ⓑ 1 · 42

Solution:
Solution

ⓐ	(11)1
The product of any number and one is the number.	11
ⓑ	1 · 42
Multiplying by one does not change the value.	42

Note:
Exercise:

Problem: Find each product:

ⓐ (19)1
ⓑ 1 · 39

Solution:

ⓐ 19
ⓑ 39

Note:
Exercise:

Problem: Find each product:

ⓐ$(24)(1)$
ⓑ $1\ \times\ 57$

Solution:

ⓐ 24
ⓑ 57

Earlier in this chapter, we learned that the Commutative Property of Addition states that changing the order of addition does not change the sum. We saw that $8+9=17$ is the same as $9+8=17$.

Is this also true for multiplication? Let’s look at a few pairs of factors.
Equation:

$$4\cdot 7=28 \qquad 7\cdot 4=28$$

Equation:

$$9\cdot 7=63 \qquad 7\cdot 9=63$$

Equation:

$$8\cdot 9=72 \qquad 9\cdot 8=72$$

When the order of the factors is reversed, the product does not change. This is called the Commutative Property of Multiplication.

Note:
Commutative Property of Multiplication
Changing the order of the factors does not change their product.
Equation:

$$a\cdot b=b\cdot a$$

Example:
Exercise:

Problem: Multiply:

ⓐ $8 \cdot 7$
ⓑ $7 \cdot 8$

Solution:
Solution

ⓐ	$8 \cdot 7$
Multiply.	56
ⓑ	$7 \cdot 8$
Multiply.	56

Changing the order of the factors does not change the product.

Note:
Exercise:

Problem: Multiply:

ⓐ $9 \cdot 6$
ⓑ $6 \cdot 9$

Solution:

54 and 54; both are the same.

Note:
Exercise:

Problem: Multiply:

ⓐ $8 \cdot 6$
ⓑ $6 \cdot 8$

Solution:

48 and 48; both are the same.

To multiply numbers with more than one digit, it is usually easier to write the numbers vertically in columns just as we did for addition and subtraction.

Equation:

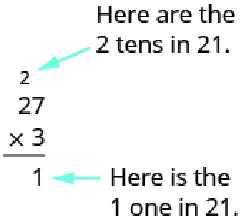
$$\begin{array}{r} 27 \\ \underline{\times 3} \end{array}$$

We start by multiplying 3 by 7.

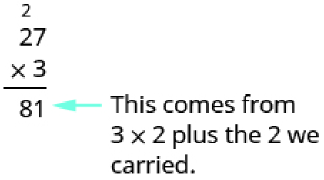
Equation:

$$3 \times 7 = 21$$

We write the 1 in the ones place of the product. We carry the 2 tens by writing 2 above the tens place.



Then we multiply the 3 by the 2, and add the 2 above the tens place to the product. So $3 \times 2 = 6$, and $6 + 2 = 8$. Write the 8 in the tens place of the product.



The product is 81.

When we multiply two numbers with a different number of digits, it's usually easier to write the smaller number on the bottom. You could write it the other way, too, but this way is easier to work with.

Example:

Exercise:

Problem: Multiply: 15 · 4.

Solution:

Solution

Write the numbers so the digits 5 and 4 line up vertically.	$\begin{array}{r} 15 \\ \times\ 4 \\ \hline \end{array}$
Multiply 4 by the digit in the ones place of 15. $4 \cdot 5 = 20$.	
Write 0 in the ones place of the product and carry the 2 tens.	$\begin{array}{r} \overset{2}{1}5 \\ \times\ 4 \\ \hline 0 \end{array}$
Multiply 4 by the digit in the tens place of 15. $4 \cdot 1 = 4$. Add the 2 tens we carried. $4 + 2 = 6$.	
Write the 6 in the tens place of the product.	$\begin{array}{r} \overset{2}{1}5 \\ \times\ 4 \\ \hline 60 \end{array}$

Note:
Exercise:

Problem: Multiply: $64 \cdot 8$.

Solution:

512

Note:
Exercise:

Problem: Multiply: $57 \cdot 6$.

Solution:

342

Example:
Exercise:

Problem: Multiply: $286 \cdot 5$.

Solution:

Solution

Write the numbers so the digits 5 and 6 line up vertically.	$\begin{array}{r} 286 \\ \times\ 5 \\ \hline \end{array}$
Multiply 5 by the digit in the ones place of 286. $5 \cdot 6 = 30$.	
Write the 0 in the ones place of the product and carry the 3 to the tens place.Multiply 5 by the digit in the tens place of 286. $5 \cdot 8 = 40$.	$\begin{array}{r} 2\overset{3}{8}6 \\ \times\ 5 \\ \hline 0 \end{array}$
Add the 3 tens we carried to get $40 + 3 = 43$. Write the 3 in the tens place of the product and carry the 4 to the hundreds place.	$\begin{array}{r} \overset{4}{2}\overset{3}{8}6 \\ \times\ 5 \\ \hline 30 \end{array}$
Multiply 5 by the digit in the hundreds place of 286. $5 \cdot 2 = 10$. Add the 4 hundreds we carried to get $10 + 4 = 14$. Write the 4 in the hundreds place of the product and the 1 to the thousands place.	$\begin{array}{r} \overset{4}{2}\overset{3}{8}6 \\ \times\ 5 \\ \hline 1{,}430 \end{array}$

Note:
Exercise:

Problem: Multiply: $347 \cdot 5$.

Solution:

$1{,}735$

Note:
Exercise:

Problem: Multiply: $462 \cdot 7$.

Solution:

$3{,}234$

When we multiply by a number with two or more digits, we multiply by each of the digits separately, working from right to left. Each separate product of the digits is called a partial product. When we write partial products, we must make sure to line up the place values.

Note:
Multiply two whole numbers to find the product.

Write the numbers so each place value lines up vertically.

Multiply the digits in each place value.

- Work from right to left, starting with the ones place in the bottom number.
    - Multiply the bottom number by the ones digit in the top number, then by the tens digit, and so on.
    - If a product in a place value is more than 9, carry to the next place value.
    - Write the partial products, lining up the digits in the place values with the numbers above.
- Repeat for the tens place in the bottom number, the hundreds place, and so on.
- Insert a zero as a placeholder with each additional partial product.

Add the partial products.

Example:
Exercise:

Problem: Multiply: $62(87)$.

Solution:
Solution

Write the numbers so each place lines up vertically.	
Start by multiplying 7 by 62. Multiply 7 by the digit in the ones place of 62. $7 \cdot 2 = 14$. Write the 4 in the ones place of the product and carry the 1 to the tens place.	
Multiply 7 by the digit in the tens place of 62. $7 \cdot 6 = 42$. Add the 1 ten we carried. $42 + 1 = 43$. Write the 3 in the tens place of the product and the 4 in the hundreds place.	
The first partial product is 434.	

Now, write a 0 under the 4 in the ones place of the next partial product as a placeholder since we now multiply the digit in the tens place of 87 by 62. Multiply 8 by the digit in the ones place of 62. $8 \cdot 2 = 16$. Write the 6 in the next place of the product, which is the tens place. Carry the 1 to the tens place.	
Multiply 8 by 6, the digit in the tens place of 62, then add the 1 ten we carried to get 49. Write the 9 in the hundreds place of the product and the 4 in the thousands place.	
The second partial product is 4960. Add the partial products.	

The product is $5{,}394$.

Note:
Exercise:

Problem: Multiply: $43(78)$.

Solution:

3,354

Note:
Exercise:

Problem: Multiply: $64(59)$.

Solution:

3,776

Example:
Exercise:

Problem: Multiply:

ⓐ $47 \cdot 10$
ⓑ $47 \cdot 100$.

Solution:
Solution

ⓐ 47 · 10.	$\begin{array}{r} 47 \\ \underline{\times 10} \\ 00 \\ \underline{470} \\ 470 \end{array}$
ⓑ 47 · 100	$\begin{array}{r} 47 \\ \underline{\times 100} \\ 00 \\ 000 \\ \underline{4700} \\ 4,700 \end{array}$

When we multiplied 47 times 10, the product was 470. Notice that 10 has one zero, and we put one zero after 47 to get the product. When we multiplied 47 times 100, the product was 4,700. Notice that 100 has two zeros and we put two zeros after 47 to get the product.

Do you see the pattern? If we multiplied 47 times 10,000, which has four zeros, we would put four zeros after 47 to get the product 470,000.

Note:
Exercise:

Problem: Multiply:

ⓐ 54 · 10
ⓑ 54 · 100.

Solution:

ⓐ 540
ⓑ 5,400

Note:
Exercise:

Problem: Multiply:

ⓐ 75 · 10
ⓑ 75 · 100.

Solution:

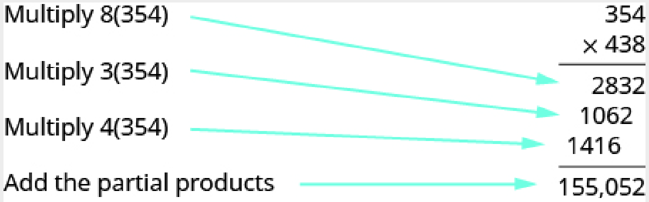
ⓐ 750
ⓑ 7,500

Example:
Exercise:

Problem: Multiply: $(354)(438)$.

Solution:
Solution

There are three digits in the factors so there will be 3 partial products. We do not have to write the 0 as a placeholder as long as we write each partial product in the correct place.



Note:
Exercise:

Problem: Multiply: $(265)(483)$.

Solution:

127,995

Note:
Exercise:

Problem: Multiply: $(823)(794)$.

Solution:

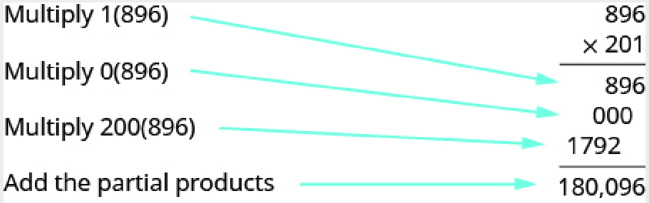
653,462

Example:
Exercise:

Problem: Multiply: $(896)201$.

Solution:
Solution

There should be 3 partial products. The second partial product will be the result of multiplying 896 by 0.



Notice that the second partial product of all zeros doesn't really affect the result. We can place a zero as a placeholder in the tens place and then proceed directly to multiplying by the 2 in the hundreds place, as shown.

Multiply by 10, but insert only one zero as a placeholder in the tens place. Multiply by 200, putting the 2 from the 12. $2 \cdot 6 = 12$ in the hundreds place.
Equation:

$$\begin{array}{r} 896 \\ \underline{\times 201} \\ 896 \\ 17920 \\ \hline 180{,}096 \end{array}$$

Note:
Exercise:

Problem: Multiply: $(718)509$.

Solution:

365,462

Note:
Exercise:

Problem: Multiply: $(627)804$.

Solution:

504,108

When there are three or more factors, we multiply the first two and then multiply their product by the next factor. For example:

to multiply	$8 \cdot 3 \cdot 2$
first multiply $8 \cdot 3$	$24 \cdot 2$
then multiply $24 \cdot 2$.	48

Translate Word Phrases to Math Notation

Earlier in this section, we translated math notation into words. Now we’ll reverse the process and translate word phrases into math notation. Some of the words that indicate multiplication are given in [link].

Operation	**Word Phrase**	**Example**	**Expression**
Multiplication	times product twice	3 times 8 the product of 3 and 8 twice 4	$3 \times 8, 3 \cdot 8, (3)(8),$ $(3)8, \text{ or } 3(8)$ $2 \cdot 4$

Example:
Exercise:

Problem: Translate and simplify: the product of 12 and 27.

Solution:
Solution

The word *product* tells us to multiply. The words *of* 12 *and* 27 tell us the two factors.

	the product of 12 and 27
Translate.	$12 \cdot 27$
Multiply.	324

Note:
Exercise:

Problem: Translate and simplify the product of 13 and 28.

Solution:

13 · 28; 364

Note:
Exercise:

Problem: Translate and simplify the product of 47 and 14.

Solution:

47 · 14; 658

Example:
Exercise:

Problem: Translate and simplify: twice two hundred eleven.

Solution:
Solution

The word *twice* tells us to multiply by 2.

	twice two hundred eleven
Translate.	2(211)

Multiply.	422

Note:
Exercise:

Problem: Translate and simplify: twice one hundred sixty-seven.

Solution:

2(167); 334

Note:
Exercise:

Problem: Translate and simplify: twice two hundred fifty-eight.

Solution:

2(258); 516

Multiply Whole Numbers in Applications

We will use the same strategy we used previously to solve applications of multiplication. First, we need to determine what we are looking for. Then we write a phrase that gives the information to find it. We then translate the phrase into math notation and simplify to get the answer. Finally, we write a sentence to answer the question.

Example:
Exercise:

Problem:

Humberto bought 4 sheets of stamps. Each sheet had 20 stamps. How many stamps did Humberto buy?

Solution:
Solution

We are asked to find the total number of stamps.

Write a phrase for the total.	the product of 4 and 20
Translate to math notation.	$4 \cdot 20$
Multiply.	$\begin{array}{r} 20 \\ \times 4 \\ \hline 80 \end{array}$
Write a sentence to answer the question.	Humberto bought 80 stamps.

Note:
Exercise:

Problem:

Valia donated water for the snack bar at her son’s baseball game. She brought 6 cases of water bottles. Each case had 24 water bottles. How many water bottles did Valia donate?

Solution:

Valia donated 144 water bottles.

Note:
Exercise:

Problem:

Vanessa brought 8 packs of hot dogs to a family reunion. Each pack has 10 hot dogs. How many hot dogs did Vanessa bring?

Solution:

Vanessa bought 80 hot dogs.

Example:
Exercise:

Problem:

When Rena cooks rice, she uses twice as much water as rice. How much water does she need to cook 4 cups of rice?

Solution:
Solution

We are asked to find how much water Rena needs.

Write as a phrase.	twice as much as 4 cups
Translate to math notation.	$2 \cdot 4$
Multiply to simplify.	8
Write a sentence to answer the question.	Rena needs 8 cups of water for cups of rice.

Note:
Exercise:

Problem:

Erin is planning her flower garden. She wants to plant twice as many dahlias as sunflowers. If she plants 14 sunflowers, how many dahlias does she need?

Solution:

Erin needs 28 dahlias.

Note:
Exercise:

Problem:

A college choir has twice as many women as men. There are 18 men in the choir. How many women are in the choir?

Solution:

There are 36 women in the choir.

Example:
Exercise:

Problem:

Van is planning to build a patio. He will have 8 rows of tiles, with 14 tiles in each row. How many tiles does he need for the patio?

Solution:
Solution

We are asked to find the total number of tiles.

Write a phrase.	the product of 8 and 14
Translate to math notation.	$8 \cdot 14$
Multiply to simplify.	$\begin{array}{r} \overset{3}{1}4 \\ \underline{\times 8} \\ 112 \end{array}$
Write a sentence to answer the question.	Van needs 112 tiles for his patio.

Note:
Exercise:

Problem:

Jane is tiling her living room floor. She will need 16 rows of tile, with 20 tiles in each row. How many tiles does she need for the living room floor?

Solution:

Jane needs 320 tiles.

Note:
Exercise:

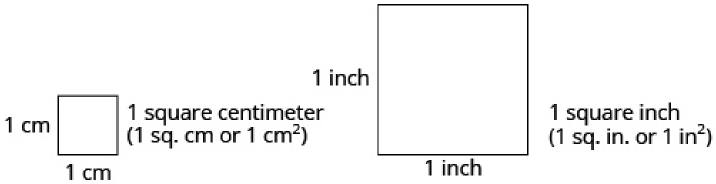
Problem:

Yousef is putting shingles on his garage roof. He will need 24 rows of shingles, with 45 shingles in each row. How many shingles does he need for the garage roof?

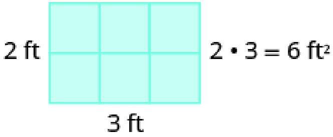
Solution:

Yousef needs 1,080 tiles.

If we want to know the size of a wall that needs to be painted or a floor that needs to be carpeted, we will need to find its **area**. The area is a measure of the amount of surface that is covered by the shape. Area is measured in square units. We often use square inches, square feet, square centimeters, or square miles to measure area. A square centimeter is a square that is one centimeter (cm.) on a side. A square inch is a square that is one inch on each side, and so on.



For a rectangular figure, the area is the product of the length and the width. [link] shows a rectangular rug with a length of 2 feet and a width of 3 feet. Each square is 1 foot wide by 1 foot long, or 1 square foot. The rug is made of 6 squares. The area of the rug is 6 square feet.



The area of a rectangle is the product of its length and its width, or 6 square feet.

Example:
Exercise:

Problem:

Jen’s kitchen ceiling is a rectangle that measures 9 feet long by 12 feet wide. What is the area of Jen’s kitchen ceiling?

Solution:
Solution

We are asked to find the area of the kitchen ceiling.

Write a phrase for the area.	the product of 9 and 12

Translate to math notation.	$9 \cdot 12$
Multiply.	$$\begin{array}{r} \overset{1}{1}2 \\ \underline{\times 9} \\ 108 \end{array}$$
Answer with a sentence.	The area of Jen's kitchen ceiling is 108 square feet.

Note:
Exercise:

Problem:

Zoila bought a rectangular rug. The rug is 8 feet long by 5 feet wide. What is the area of the rug?

Solution:

The area of the rug is 40 square feet.

Note:
Exercise:

Problem:

Rene’s driveway is a rectangle 45 feet long by 20 feet wide. What is the area of the driveway?

Solution:

The area of the driveway is 900 square feet

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Multiplying Whole Numbers
- Multiplication with Partial Products
- Example of Multiplying by Whole Numbers

Key Concepts

Operation	Notation	Expression	Read as	Result
Multiplication	$\times$ $\cdot$ $(\,)$	3×8 $3 \cdot 8$ $3(8)$	three times eight	the product of 3 and 8

- **Multiplication Property of Zero**
  - The product of any number and 0 is 0.
    $a \cdot 0 = 0$
    $0 \cdot a = 0$
- **Identity Property of Multiplication**
  - The product of any number and 1 is the number.
    $1 \cdot a = a$
    $a \cdot 1 = a$
- **Commutative Property of Multiplication**
  - Changing the order of the factors does not change their product.
    $a \cdot b = b \cdot a$
- **Multiply two whole numbers to find the product.**

  Write the numbers so each place value lines up vertically.
  Multiply the digits in each place value.
  Work from right to left, starting with the ones place in the bottom number.
  Multiply the bottom number by the ones digit in the top number, then by the tens digit, and so on.
  If a product in a place value is more than 9, carry to the next place value.
  Write the partial products, lining up the digits in the place values with the numbers above. Repeat for the tens place in the bottom number, the hundreds place, and so on.
  Insert a zero as a placeholder with each additional partial product.
  Add the partial products.

Practice Makes Perfect

Use Multiplication Notation

In the following exercises, translate from math notation to words.

Exercise:

Problem: 4×7

Solution:

four times seven; the product of four and seven

Exercise:

Problem: 8×6

Exercise:

Problem: $5 \cdot 12$

Solution:

five times twelve; the product of five and twelve

Exercise:

Problem: $3 \cdot 9$

Exercise:

Problem: $(10)(25)$

Solution:

ten times twenty-five; the product of ten and twenty-five

Exercise:

Problem: $(20)(15)$

Exercise:

Problem: $42(33)$

Solution:

forty-two times thirty-three; the product of forty-two and thirty-three

Exercise:

Problem: $39(64)$

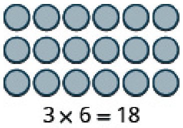
Model Multiplication of Whole Numbers

In the following exercises, model the multiplication.

Exercise:

Problem: 3×6

Solution:



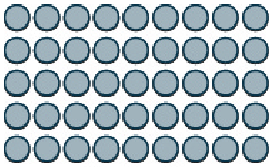
Exercise:

Problem: 4×5

Exercise:

Problem: 5×9

Solution:



5 × 9 = 45

Exercise:

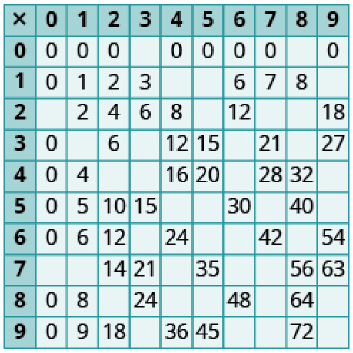
Problem: 3×9

Multiply Whole Numbers

In the following exercises, fill in the missing values in each chart.

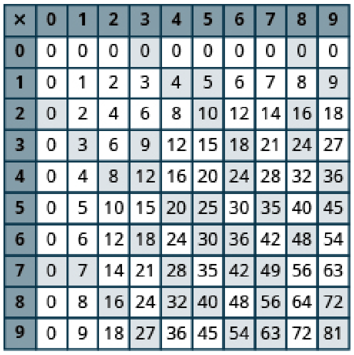
Exercise:

Problem:



×	0	1	2	3	4	5	6	7	8	9
0	0	0	0		0	0	0	0		0
1	0	1	2	3			6	7	8	
2		2	4	6	8		12			18
3	0		6		12	15		21		27
4	0	4			16	20		28	32	
5	0	5	10	15			30		40	
6	0	6	12		24			42		54
7			14	21		35			56	63
8	0	8		24			48		64	
9	0	9	18		36	45			72	

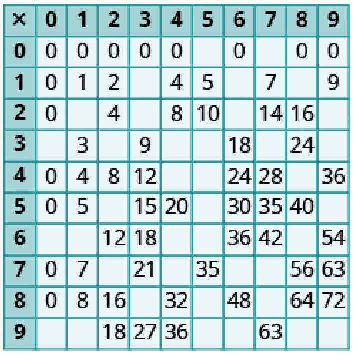
Solution:



×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Exercise:

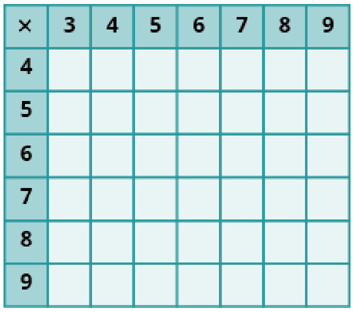
Problem:



×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0		0		0	0
1	0	1	2		4	5		7		9
2	0		4		8	10		14	16	
3		3		9			18		24	
4	0	4	8	12			24	28		36
5	0	5		15	20		30	35	40	
6			12	18			36	42		54
7	0	7		21		35			56	63
8	0	8	16		32		48		64	72
9			18	27	36			63		

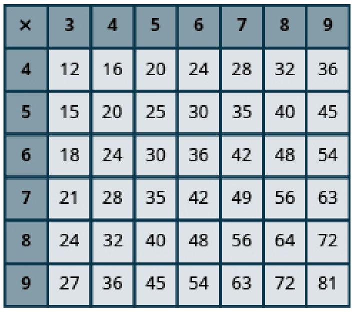
Exercise:

Problem:



×	3	4	5	6	7	8	9
4							
5							
6							
7							
8							
9							

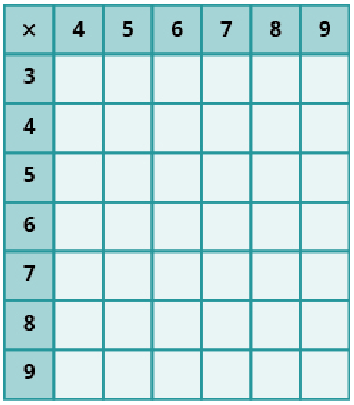
Solution:



×	3	4	5	6	7	8	9
4	12	16	20	24	28	32	36
5	15	20	25	30	35	40	45
6	18	24	30	36	42	48	54
7	21	28	35	42	49	56	63
8	24	32	40	48	56	64	72
9	27	36	45	54	63	72	81

Exercise:

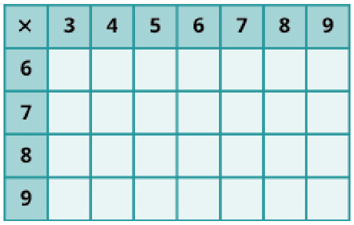
Problem:



×	4	5	6	7	8	9
3						
4						
5						
6						
7						
8						
9						

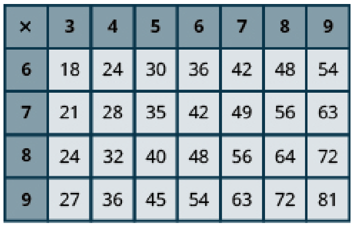
Exercise:

Problem:



×	3	4	5	6	7	8	9
6							
7							
8							
9							

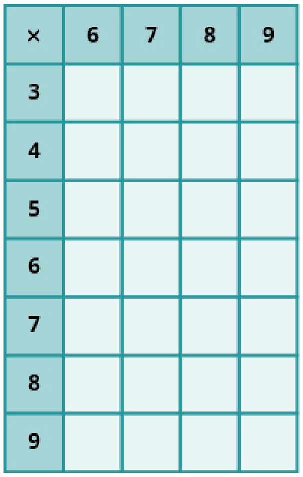
Solution:



×	3	4	5	6	7	8	9
6	18	24	30	36	42	48	54
7	21	28	35	42	49	56	63
8	24	32	40	48	56	64	72
9	27	36	45	54	63	72	81

Exercise:

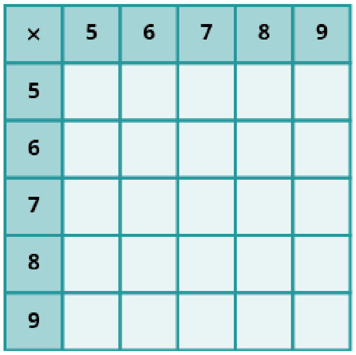
Problem:



×	6	7	8	9
3				
4				
5				
6				
7				
8				
9				

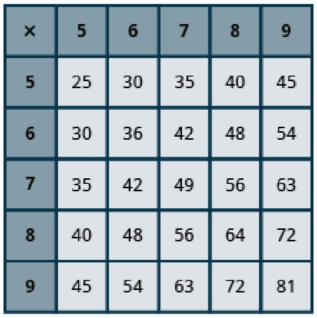
Exercise:

Problem:



×	5	6	7	8	9
5					
6					
7					
8					
9					

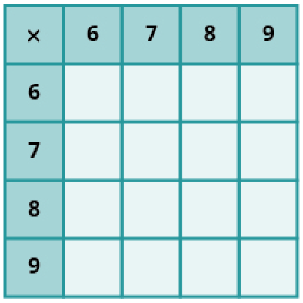
Solution:



×	5	6	7	8	9
5	25	30	35	40	45
6	30	36	42	48	54
7	35	42	49	56	63
8	40	48	56	64	72
9	45	54	63	72	81

Exercise:

Problem:



×	6	7	8	9
6				
7				
8				
9				

In the following exercises, multiply.

Exercise:

Problem: $0 \cdot 15$

Solution:

0

Exercise:

Problem: $0 \cdot 41$

Exercise:

Problem: $(99)0$

Solution:

0

Exercise:

Problem: $(77)0$

Exercise:

Problem: $1 \cdot 43$

Solution:

43

Exercise:

Problem: $1 \cdot 34$

Exercise:

Problem: $(28)1$

Solution:

28

Exercise:

Problem: $(65)1$

Exercise:

Problem: $1(240{,}055)$

Solution:

240,055

Exercise:

Problem: $1(189{,}206)$

Exercise:

Problem:

ⓐ $7 \cdot 6$
ⓑ $6 \cdot 7$

Solution:

ⓐ 42
ⓑ 42

Exercise:

Problem:

ⓐ 8×9
ⓑ 9×8

Exercise:

Problem: $(79)(5)$

Solution:

395

Exercise:

Problem: $(58)(4)$

Exercise:

Problem: $275 \cdot 6$

Solution:

1,650

Exercise:

Problem: $638 \cdot 5$

Exercise:

Problem: $3{,}421 \times 7$

Solution:

23,947

Exercise:

Problem: $9{,}143 \times 3$

Exercise:

Problem: $52(38)$

Solution:

1,976

Exercise:

Problem: $37(45)$

Exercise:

Problem: $96 \cdot 73$

Solution:

7,008

Exercise:

Problem: $89 \cdot 56$

Exercise:

Problem: 27×85

Solution:

2,295

Exercise:

Problem: 53×98

Exercise:

Problem: $23 \cdot 10$

Solution:

230

Exercise:

Problem: $19 \cdot 10$

Exercise:

Problem: $(100)(36)$

Solution:

360

Exercise:

Problem: $(100)(25)$

Exercise:

Problem: $1{,}000(88)$

Solution:

88,000

Exercise:

Problem: $1{,}000(46)$

Exercise:

Problem: $50 \ \times \ 1{,}000{,}000$

Solution:

50,000,000

Exercise:

Problem: $30 \ \times \ 1{,}000{,}000$

Exercise:

Problem: $247 \ \times \ 139$

Solution:

34,333

Exercise:

Problem: 156×328

Exercise:

Problem: $586(721)$

Solution:

422,506

Exercise:

Problem: $472(855)$

Exercise:

Problem: $915 \cdot 879$

Solution:

804,285

Exercise:

Problem: $968 \cdot 926$

Exercise:

Problem: $(104)(256)$

Solution:

26,624

Exercise:

Problem: $(103)(497)$

Exercise:

Problem: $348(705)$

Solution:

245,340

Exercise:

Problem: $485(602)$

Exercise:

Problem: $2{,}719\ \times\ 543$

Solution:

1,476,417

Exercise:

Problem: $3{,}581\ \times\ 724$

Translate Word Phrases to Math Notation

In the following exercises, translate and simplify.

Exercise:

Problem: the product of 18 and 33

Solution:

18 · 33; 594

Exercise:

Problem: the product of 15 and 22

Exercise:

Problem: fifty-one times sixty-seven

Solution:

51(67); 3,417

Exercise:

Problem: forty-eight times seventy-one

Exercise:

Problem: twice 249

Solution:

2(249); 498

Exercise:

Problem: twice 589

Exercise:

Problem: ten times three hundred seventy-five

Solution:

10(375); 3,750

Exercise:

Problem: ten times two hundred fifty-five

Mixed Practice

In the following exercises, simplify.

Exercise:

Problem: 38×37

Solution:

1,406

Exercise:

Problem: 86×29

Exercise:

Problem: $415 - 267$

Solution:

148

Exercise:

Problem: $341 - 285$

Exercise:

Problem: $6{,}251 + 4{,}749$

Solution:

11,000

Exercise:

Problem: $3{,}816 + 8{,}184$

Exercise:

Problem: $(56)(204)$

Solution:

11,424

Exercise:

Problem: $(77)(801)$

Exercise:

Problem: $947 \cdot 0$

Solution:

0

Exercise:

Problem: $947 + 0$

Exercise:

Problem: $15,382 + 1$

Solution:

15,383

Exercise:

Problem: $15,382 \cdot 1$

In the following exercises, translate and simplify.

Exercise:

Problem: the difference of 50 and 18

Solution:

50 − 18; 32

Exercise:

Problem: the difference of 90 and 66

Exercise:

Problem: twice 35

Solution:

2(35); 70

Exercise:

Problem: twice 140

Exercise:

Problem: 20 more than 980

Solution:

20 + 980; 1,000

Exercise:

Problem: 65 more than 325

Exercise:

Problem: the product of 12 and 875

Solution:

12(875); 10,500

Exercise:

Problem: the product of 15 and 905

Exercise:

Problem: subtract 74 from 89

Solution:

89 − 74; 15

Exercise:

Problem: subtract 45 from 99

Exercise:

Problem: the sum of 3,075 and 950

Solution:

3,075 + 950; 4,025

Exercise:

Problem: the sum of 6,308 and 724

Exercise:

Problem: 366 less than 814

Solution:

814 − 366; 448

Exercise:

Problem: 388 less than 925

Multiply Whole Numbers in Applications

In the following exercises, solve.

Exercise:

Problem:

Party supplies Tim brought 9 six-packs of soda to a club party. How many cans of soda did Tim bring?

Solution:

Tim brought 54 cans of soda to the party.

Exercise:

Problem:

Sewing Kanisha is making a quilt. She bought 6 cards of buttons. Each card had four buttons on it. How many buttons did Kanisha buy?

Exercise:

Problem:

Field trip Seven school busses let off their students in front of a museum in Washington, DC. Each school bus had 44 students. How many students were there?

Solution:

There were 308 students.

Exercise:

Problem:

Gardening Kathryn bought 8 flats of impatiens for her flower bed. Each flat has 24 flowers. How many flowers did Kathryn buy?

Exercise:

Problem:

Charity Rey donated 15 twelve-packs of t-shirts to a homeless shelter. How many t-shirts did he donate?

Solution:

Rey donated 180 t-shirts.

Exercise:

Problem:

School There are 28 classrooms at Anna C. Scott elementary school. Each classroom has 26 student desks. What is the total number of student desks?

Exercise:

Problem:

Recipe Stephanie is making punch for a party. The recipe calls for twice as much fruit juice as club soda. If she uses 10 cups of club soda, how much fruit juice should she use?

Solution:

Stephanie should use 20 cups of fruit juice.

Exercise:

Problem:

Gardening Hiroko is putting in a vegetable garden. He wants to have twice as many lettuce plants as tomato plants. If he buys 12 tomato plants, how many lettuce plants should he get?

Exercise:

Problem:

Government The United States Senate has twice as many senators as there are states in the United States. There are 50 states. How many senators are there in the United States Senate?

Solution:

There are 100 senators in the U.S. senate.

Exercise:

Problem:

Recipe Andrea is making potato salad for a buffet luncheon. The recipe says the number of servings of potato salad will be twice the number of pounds of potatoes. If she buys 30 pounds of potatoes, how many servings of potato salad will there be?

Exercise:

Problem:

Painting Jane is painting one wall of her living room. The wall is rectangular, 13 feet wide by 9 feet high. What is the area of the wall?

Solution:

The area of the wall is 117 square feet.

Exercise:

Problem:

Home décor Shawnte bought a rug for the hall of her apartment. The rug is 3 feet wide by 18 feet long. What is the area of the rug?

Exercise:

Problem:

Room size The meeting room in a senior center is rectangular, with length 42 feet and width 34 feet. What is the area of the meeting room?

Solution:

The area of the room is 1,428 square feet.

Exercise:

Problem:

Gardening June has a vegetable garden in her yard. The garden is rectangular, with length 23 feet and width 28 feet. What is the area of the garden?

Exercise:

Problem:

NCAA basketball According to NCAA regulations, the dimensions of a rectangular basketball court must be 94 feet by 50 feet. What is the area of the basketball court?

Solution:

The area of the court is 4,700 square feet.

Exercise:

Problem:

NCAA football According to NCAA regulations, the dimensions of a rectangular football field must be 360 feet by 160 feet. What is the area of the football field?

Everyday Math

Exercise:

Problem:

Stock market Javier owns 300 shares of stock in one company. On Tuesday, the stock price rose $12 per share. How much money did Javier's portfolio gain?

Solution:

Javier's portfolio gained $3,600.

Exercise:

Problem:

Salary Carlton got a $200 raise in each paycheck. He gets paid 24 times a year. How much higher is his new annual salary?

Writing Exercises

Exercise:

Problem:

How confident do you feel about your knowledge of the multiplication facts? If you are not fully confident, what will you do to improve your skills?

Solution:

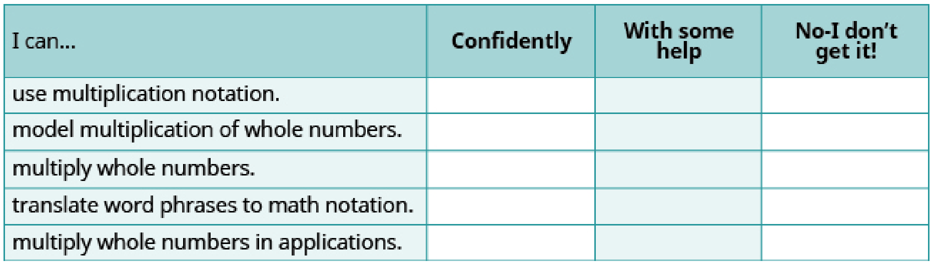
Answers will vary.

Exercise:

Problem: How have you used models to help you learn the multiplication facts?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
use multiplication notation.			
model multiplication of whole numbers.			
multiply whole numbers.			
translate word phrases to math notation.			
multiply whole numbers in applications.			

ⓑ On a scale of 1–10, how would you rate your mastery of this section in light of your responses on the checklist? How can you improve this?

Glossary

product

The product is the result of multiplying two or more numbers.

Divide Whole Numbers

By the end of this section, you will be able to:

- Use division notation
- Model division of whole numbers
- Divide whole numbers
- Translate word phrases to math notation
- Divide whole numbers in applications

Note:
Before you get started, take this readiness quiz.

1. Multiply: $27 \cdot 3$.
   If you missed this problem, review [link].
2. Subtract: $43 - 26$.
   If you missed this problem, review [link]
3. Multiply: $62(87)$.
   If you missed this problem, review [link].

Use Division Notation

So far we have explored addition, subtraction, and multiplication. Now let's consider division. Suppose you have the 12 cookies in [link] and want to package them in bags with 4 cookies in each bag. How many bags would we need?

You might put 4 cookies in first bag, 4 in the second bag, and so on until you run out of cookies. Doing it this way, you would fill 3 bags.

In other words, starting with the 12 cookies, you would take away, or subtract, 4 cookies at a time. Division is a way to represent repeated subtraction just as multiplication represents repeated addition.

Instead of subtracting 4 repeatedly, we can write

Equation:

$$12 \div 4$$

We read this as *twelve divided by four* and the result is the **quotient** of 12 and 4. The quotient is 3 because we can subtract 4 from 12 exactly 3 times. We call the number being divided the **dividend** and the number dividing it the **divisor**. In this case, the dividend is 12 and the divisor is 4.

In the past you may have used the notation $4\overline{)12}$, but this division also can be written as $12 \div 4,\ 12/4,\ \frac{12}{4}$. In each case the 12 is the dividend and the 4 is the divisor.

Note:
Operation Symbols for Division
To represent and describe division, we can use symbols and words.

Operation	**Notation**	**Expression**	**Read as**	**Result**
Division	$\div$ $\frac{a}{b}$ $b\overline{)a}$ a/b	$12 \div 4$ $\frac{12}{4}$ $4\overline{)12}$ $12/4$	Twelve divided by four	the quotient of 12 and 4

Division is performed on two numbers at a time. When translating from math notation to English words, or English words to math notation, look for the words *of* and *and* to identify the numbers.

Example:
Exercise:

Problem: Translate from math notation to words.

ⓐ $64 \div 8$ ⓑ $\frac{42}{7}$ ⓒ $4\overline{)28}$

Solution:
Solution

- ⓐ We read this as *sixty-four divided by eight* and the result is *the quotient of sixty-four and eight*.
- ⓑ We read this as *forty-two divided by seven* and the result is *the quotient of forty-two and seven*.
- ⓒ We read this as *twenty-eight divided by four* and the result is *the quotient of twenty-eight and four*.

Note:
Exercise:

Problem: Translate from math notation to words:

ⓐ $84 \div 7$ ⓑ $\frac{18}{6}$ ⓒ $8\overline{)24}$

Solution:

- ⓐ eighty-four divided by seven; the quotient of eighty-four and seven
- ⓑ eighteen divided by six; the quotient of eighteen and six.
- ⓒ twenty-four divided by eight; the quotient of twenty-four and eight

Note:
Exercise:

Problem: Translate from math notation to words:

ⓐ $72 \div 9$ ⓑ $\frac{21}{3}$ ⓒ $6\overline{)54}$

Solution:

- ⓐ seventy-two divided by nine; the quotient of seventy-two and nine
- ⓑ twenty-one divided by three; the quotient of twenty-one and three
- ⓒ fifty-four divided by six; the quotient of fifty-four and six

Model Division of Whole Numbers

As we did with multiplication, we will model division using counters. The operation of division helps us organize items into equal groups as we start with the number of items in the dividend and subtract the number in the divisor repeatedly.

Note: Doing the Manipulative Mathematics activity Model Division of Whole Numbers will help you develop a better understanding of dividing whole numbers.

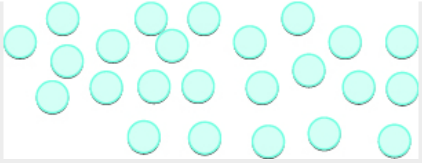
Example:
Exercise:

Problem: Model the division: $24 \div 8$.

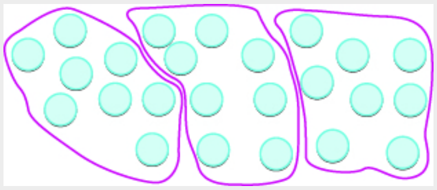
Solution:
Solution

To find the quotient $24 \div 8$, we want to know how many groups of 8 are in 24.

Model the dividend. Start with 24 counters.



The divisor tell us the number of counters we want in each group. Form groups of 8 counters.



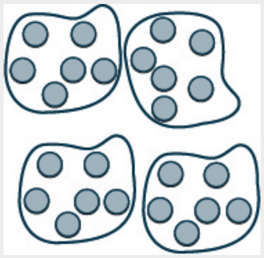
Count the number of groups. There are 3 groups.

$24 \div 8 = 3$

Note:
Exercise:

Problem: Model: $24 \div 6$.

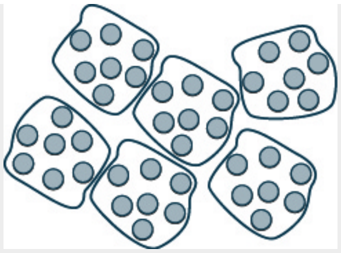
Solution:



Note:
Exercise:

Problem: Model: $42 \div 7$.

Solution:



Divide Whole Numbers

We said that addition and subtraction are inverse operations because one undoes the other. Similarly, division is the inverse operation of multiplication. We know $12 \div 4 = 3$ because $3 \cdot 4 = 12$. Knowing all the multiplication number facts is very important when doing division.

We check our answer to division by multiplying the quotient by the divisor to determine if it equals the dividend. In [link], we know $24 \div 8 = 3$ is correct because $3 \cdot 8 = 24$.

Example:
Exercise:

Problem: Divide. Then check by multiplying. ⓐ $42 \div 6$ ⓑ $\frac{72}{9}$ ⓒ $7\overline{)63}$

Solution:
Solution

-
ⓐ	
	mtd
Divide 42 by 6.	7
Check by multiplying. $7 \cdot 6$	
42✓	

-
ⓑ	

	$\frac{72}{9}$
Divide 72 by 9.	8
Check by multiplying. $8 \cdot 9$	
$72\checkmark$	

ⓒ	
	$7\overline{)63}$
Divide 63 by 7.	9
Check by multiplying. $9 \cdot 7$	
$63\checkmark$	

Note:
Exercise:

Problem: Divide. Then check by multiplying:

ⓐ $54 \div 6$ ⓑ $\frac{27}{9}$

Solution:

ⓐ 9 ⓑ 3

Note:
Exercise:

Problem: Divide. Then check by multiplying:

ⓐ $\frac{36}{9}$ ⓑ $8\overline{)40}$

Solution:

ⓐ 4 ⓑ 5

What is the quotient when you divide a number by itself?
Equation:

$$\frac{15}{15} = 1 \text{ because } 1 \cdot 15 = 15$$

Dividing any number $(\text{except } 0)$ by itself produces a quotient of 1. Also, any number divided by 1 produces a quotient of the number. These two ideas are stated in the Division Properties of One.

Note:
Division Properties of One

Any number (except 0) divided by itself is one.	$a \div a = 1$
Any number divided by one is the same number.	$a \div \mathit{1} = \mathrm{a}$

Example:
Exercise:

Problem: Divide. Then check by multiplying:

ⓐ $11 \div 11$
ⓑ $\frac{19}{1}$
ⓒ $1\overline{)7}$

Solution:
Solution

-

ⓐ	
	$11 \div 11$
A number divided by itself is 1.	1
Check by multiplying. $1 \cdot 11$	

11✓	

ⓑ	
	$\frac{19}{1}$
A number divided by 1 equals itself.	19
Check by multiplying. $19 \cdot 1$	
19✓	

ⓒ	
	$1\overline{)7}$
A number divided by 1 equals itself.	7
Check by multiplying. $7 \cdot 1$	
7✓	

Note:
Exercise:

Problem: Divide. Then check by multiplying:

ⓐ $14 \div 14$ ⓑ $\frac{27}{1}$

Solution:

ⓐ 1
ⓑ 27

Note:
Exercise:

Problem: Divide. Then check by multiplying:

ⓐ $\frac{16}{1}$ ⓑ $1\overline{)4}$

Solution:

ⓐ 16
ⓑ 4

Suppose we have \$0, and want to divide it among 3 people. How much would each person get? Each person would get \$0. Zero divided by any number is 0.

Now suppose that we want to divide \$10 by 0. That means we would want to find a number that we multiply by 0 to get 10. This cannot happen because 0 times any number is 0. Division by zero is said to be *undefined*.

These two ideas make up the Division Properties of Zero.

Note:
Division Properties of Zero

Zero divided by any number is 0.	$0 \div a = 0$
Dividing a number by zero is undefined.	$a \div 0$ undefined

Another way to explain why division by zero is undefined is to remember that division is really repeated subtraction. How many times can we take away 0 from 10? Because subtracting 0 will never change the total, we will never get an answer. So we cannot divide a number by 0.

Example:
Exercise:

Problem: Divide. Check by multiplying: ⓐ $0 \div 3$ ⓑ $10/0$.

Solution:
Solution

-

ⓐ	
	$0 \div 3$
Zero divided by any number is zero.	0
Check by multiplying. $0 \cdot 3$	
0✓	

ⓑ	
	$10/0$
Division by zero is undefined.	undefined

Note:
Exercise:

Problem: Divide. Then check by multiplying:

ⓐ $0 \div 2$ ⓑ $17/0$

Solution:

ⓐ 0 ⓑ undefined

Note:
Exercise:

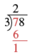
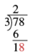
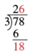
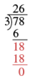
Problem: Divide. Then check by multiplying:

ⓐ $0 \div 6$ ⓑ $13/0$

Solution:

ⓐ 0 ⓑ undefined

When the divisor or the dividend has more than one digit, it is usually easier to use the $4\overline{)12}$ notation. This process is called long division. Let's work through the process by dividing 78 by 3.

Divide the first digit of the dividend, 7, by the divisor, 3.	
The divisor 3 can go into 7 two times since $2\times3=6$. Write the 2 above the 7 in the quotient.	
Multiply the 2 in the quotient by 2 and write the product, 6, under the 7.	
Subtract that product from the first digit in the dividend. Subtract $7-6$. Write the difference, 1, under the first digit in the dividend.	
Bring down the next digit of the dividend. Bring down the 8.	
Divide 18 by the divisor, 3. The divisor 3 goes into 18 six times.	
Write 6 in the quotient above the 8.	
Multiply the 6 in the quotient by the divisor and write the product, 18, under the dividend. Subtract 18 from 18.	

We would repeat the process until there are no more digits in the dividend to bring down. In this problem, there are no more digits to bring down, so the division is finished.

Equation:

$$\text{So } 78\div3=26.$$

Check by multiplying the quotient times the divisor to get the dividend. Multiply $26\ \times\ 3$ to make sure that product equals the dividend, 78.

Equation:

$$\begin{array}{r}\overset{1}{2}6\\ \times3\\ \hline 78\checkmark\end{array}$$

It does, so our answer is correct.

Note:
Divide whole numbers.

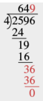
Divide the first digit of the dividend by the divisor.
If the divisor is larger than the first digit of the dividend, divide the first two digits of the dividend by the divisor, and so on.
Write the quotient above the dividend.
Multiply the quotient by the divisor and write the product under the dividend.
Subtract that product from the dividend.
Bring down the next digit of the dividend.
Repeat from Step 1 until there are no more digits in the dividend to bring down.
Check by multiplying the quotient times the divisor.

Example:
Exercise:

Problem: Divide $2{,}596 \div 4$. Check by multiplying:

Solution:
Solution

Let's rewrite the problem to set it up for long division.	$4\overline{)2596}$
Divide the first digit of the dividend, 2, by the divisor, 4.	$4\overline{)2596}$
Since 4 does not go into 2, we use the first two digits of the dividend and divide 25 by 4. The divisor 4 goes into 25 six times.	
We write the 6 in the quotient above the 5.	$\begin{array}{r} 6 \\ 4\overline{)2596} \end{array}$
Multiply the 6 in the quotient by the divisor 4 and write the product, 24, under the first two digits in the dividend.	$\begin{array}{r} 6 \\ 4\overline{)2596} \\ \underline{24} \end{array}$
Subtract that product from the first two digits in the dividend. Subtract $25-24$. Write the difference, 1, under the second digit in the dividend.	$\begin{array}{r} 6 \\ 4\overline{)2596} \\ \underline{24} \\ 1 \end{array}$
Now bring down the 9 and repeat these steps. There are 4 fours in 19. Write the 4 over the 9. Multiply the 4 by 4 and subtract this product from 19.	$\begin{array}{r} 64 \\ 4\overline{)2596} \\ \underline{24} \\ 19 \\ \underline{16} \\ 3 \end{array}$

Bring down the 6 and repeat these steps. There are 9 fours in 36. Write the 9 over the 6. Multiply the 9 by 4 and subtract this product from 36.	
So $2{,}596 \div 4 = 649$.	
Check by multiplying. $$\begin{array}{r} \overset{1\,3}{6}49 \\ \times \quad 4 \\ \hline 2{,}596\checkmark \end{array}$$	

It equals the dividend, so our answer is correct.

Note:
Exercise:

Problem: Divide. Then check by multiplying: $2{,}636 \div 4$

Solution:

659

Note:
Exercise:

Problem: Divide. Then check by multiplying: $2{,}716 \div 4$

Solution:

679

Example:
Exercise:

Problem: Divide $4{,}506 \div 6$. Check by multiplying:

Solution:
Solution

Let's rewrite the problem to set it up for long division.	$6\overline{)4506}$
First we try to divide 6 into 4.	$6\overline{)4506}$
Since that won't work, we try 6 into 45. There are 7 sixes in 45. We write the 7 over the 5.	$\begin{array}{r}7\\6\overline{)4506}\end{array}$
Multiply the 7 by 6 and subtract this product from 45.	$\begin{array}{r}7\\6\overline{)4506}\\\underline{42}\\3\end{array}$
Now bring down the 0 and repeat these steps. There are 5 sixes in 30. Write the 5 over the 0. Multiply the 5 by 6 and subtract this product from 30.	$\begin{array}{r}75\\6\overline{)4506}\\\underline{42}\\30\\\underline{30}\\0\end{array}$
Now bring down the 6 and repeat these steps. There is 1 six in 6. Write the 1 over the 6. Multiply 1 by 6 and subtract this product from 6.	$\begin{array}{r}751\\6\overline{)4506}\\\underline{42}\\30\\\underline{30}\\06\\\underline{6}\\0\end{array}$
Check by multiplying. $\begin{array}{r}\overset{3}{7}51\\\times\quad 6\\\hline 4{,}506\end{array}$ ✓	

It equals the dividend, so our answer is correct.

Note:
Exercise:

Problem: Divide. Then check by multiplying: 4,305 ÷ 5.

Solution:

861

Note:
Exercise:

Problem: Divide. Then check by multiplying: $3,906 \div 6$.

Solution:

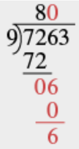
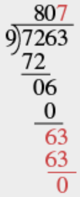
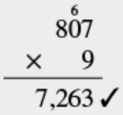
651

Example:
Exercise:

Problem: Divide $7,263 \div 9$. Check by multiplying.

Solution:
Solution

Let's rewrite the problem to set it up for long division.	$9\overline{)7263}$
First we try to divide 9 into 7.	$9\overline{)7263}$
Since that won't work, we try 9 into 72. There are 8 nines in 72. We write the 8 over the 2.	$\begin{array}{r} 8 \\ 9\overline{)7263} \end{array}$
Multiply the 8 by 9 and subtract this product from 72.	$\begin{array}{r} 8 \\ 9\overline{)7263} \\ \underline{72} \\ 0 \end{array}$
Now bring down the 6 and repeat these steps. There are 0 nines in 6. Write the 0 over the 6. Multiply the 0 by 9 and subtract this product from 6.	

	
Now bring down the 3 and repeat these steps. There are 7 nines in 63. Write the 7 over the 3. Multiply the 7 by 9 and subtract this product from 63.	
Check by multiplying. 	

It equals the dividend, so our answer is correct.

Note:
Exercise:

Problem: Divide. Then check by multiplying: 4,928 ÷ 7.

Solution:

704

Note:
Exercise:

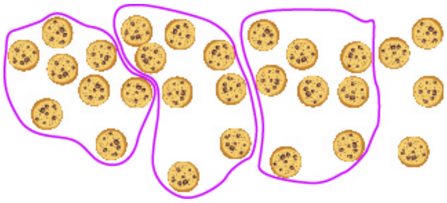
Problem: Divide. Then check by multiplying: 5,663 ÷ 7.

Solution:

809

So far all the division problems have worked out evenly. For example, if we had 24 cookies and wanted to make bags of 8 cookies, we would have 3 bags. But what if there were 28 cookies and we wanted to make bags of 8? Start with the 28 cookies as shown in [link].

Try to put the cookies in groups of eight as in [link].



There are 3 groups of eight cookies, and 4 cookies left over. We call the 4 cookies that are left over the remainder and show it by writing R4 next to the 3. (The R stands for remainder.)

To check this division we multiply 3 times 8 to get 24, and then add the remainder of 4.

Equation:

$$\begin{array}{r} 3 \\ \underline{\times 8} \\ 24 \\ \underline{+4} \\ 28 \end{array}$$

Example:

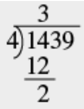
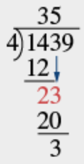
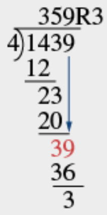
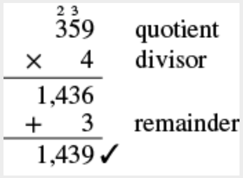
Exercise:

Problem: Divide $1{,}439 \div 4$. Check by multiplying.

Solution:

Solution

Let's rewrite the problem to set it up for long division.	$4\overline{)1439}$
First we try to divide 4 into 1. Since that won't work, we try 4 into 14. There are 3 fours in 14. We write the 3 over the 4.	$\begin{array}{r} 3 \\ 4\overline{)1439} \end{array}$

Multiply the 3 by 4 and subtract this product from 14.	
Now bring down the 3 and repeat these steps. There are 5 fours in 23. Write the 5 over the 3. Multiply the 5 by 4 and subtract this product from 23.	
Now bring down the 9 and repeat these steps. There are 9 fours in 39. Write the 9 over the 9. Multiply the 9 by 4 and subtract this product from 39. There are no more numbers to bring down, so we are done. The remainder is 3.	
Check by multiplying.  	

So 1,439 ÷ 4 is 359 with a remainder of 3. Our answer is correct.

Note:
Exercise:

Problem: Divide. Then check by multiplying: 3,812 ÷ 8.

Solution:

476 with a remainder of 4

Note:
Exercise:

Problem: Divide. Then check by multiplying: 4,319 ÷ 8.

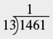
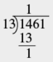
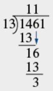
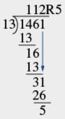
Solution:

539 with a remainder of 7

Example:
Exercise:

Problem: Divide and then check by multiplying: $1,461 \div 13$.

Solution:
Solution

Let's rewrite the problem to set it up for long division.	$13\overline{)1,461}$
First we try to divide 13 into 1. Since that won't work, we try 13 into 14. There is 1 thirteen in 14. We write the 1 over the 4.	
Multiply the 1 by 13 and subtract this product from 14.	
Now bring down the 6 and repeat these steps. There is 1 thirteen in 16. Write the 1 over the 6. Multiply the 1 by 13 and subtract this product from 16.	
Now bring down the 1 and repeat these steps. There are 2 thirteens in 31. Write the 2 over the 1. Multiply the 2 by 13 and subtract this product from 31. There are no more numbers to bring down, so we are done. The remainder is 5. $1,462 \div 13$ is 112 with a remainder of 5.	
Check by multiplying. $\begin{array}{rl} 112 & \text{quotient} \\ \underline{\times\ \ 13} & \text{divisor} \\ 336 & \\ 1,120 & \\ \underline{+\quad 5} & \text{remainder} \\ 1,461 & \checkmark \end{array}$	

Our answer is correct.

Note:
Exercise:

Problem: Divide. Then check by multiplying: $1{,}493 \div 13$.

Solution:

114 R11

Note:
Exercise:

Problem: Divide. Then check by multiplying: $1{,}461 \div 12$.

Solution:

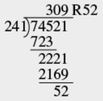
121 R9

Example:
Exercise:

Problem: Divide and check by multiplying: $74{,}521 \div 241$.

Solution:
Solution

Let's rewrite the problem to set it up for long division.	$241\overline{)74{,}521}$
First we try to divide 241 into 7. Since that won’t work, we try 241 into 74. That still won’t work, so we try 241 into 745. Since 2 divides into 7 three times, we try 3. Since $3 \times 241 = 723$, we write the 3 over the 5 in 745. Note that 4 would be too large because $4 \times 241 = 964$, which is greater than 745.	
Multiply the 3 by 241 and subtract this product from 745.	$\begin{array}{r} 3 \\ 241\overline{)74521} \\ \underline{723} \\ 22 \end{array}$
Now bring down the 2 and repeat these steps. 241 does not divide into 222. We write a 0 over the 2 as a placeholder and then continue.	$\begin{array}{r} 3 \\ 241\overline{)74521} \\ \underline{723} \\ 22 \end{array}$

Now bring down the 1 and repeat these steps. Try 9. Since $9 \times 241 = 2{,}169$, we write the 9 over the 1. Multiply the 9 by 241 and subtract this product from 2,221.	
There are no more numbers to bring down, so we are finished. The remainder is 52. So $74{,}521 \div 241$ is 309 with a remainder of 52.	
Check by multiplying. $\begin{array}{rl} \scriptstyle 3 & \\ 309 & \text{quotient} \\ \times\ 241 & \text{divisor} \\ \hline 309 & \\ 12{,}360 & \\ 61{,}800 & \\ \hline 72{,}469 & \\ +\quad 52 & \text{remainder} \\ \hline 74{,}521 & \checkmark \end{array}$	

Sometimes it might not be obvious how many times the divisor goes into digits of the dividend. We will have to guess and check numbers to find the greatest number that goes into the digits without exceeding them.

Note:
Exercise:

Problem: Divide. Then check by multiplying: $78{,}641 \div 256$.

Solution:

307 R49

Note:
Exercise:

Problem: Divide. Then check by multiplying: $76{,}461 \div 248$.

Solution:

308 R77

Translate Word Phrases to Math Notation

Earlier in this section, we translated math notation for division into words. Now we’ll translate word phrases into math notation. Some of the words that indicate division are given in [link].

Operation	**Word Phrase**	**Example**	**Expression**
Division	divided by quotient of divided into	12 divided by 4 the quotient of 12 and 4 4 divided into 12	$12 \div 4$ $\frac{12}{4}$ $12/4$ $4\overline{)12}$

Example:
Exercise:

Problem: Translate and simplify: the quotient of 51 and 17.

Solution:
Solution

The word *quotient* tells us to divide.

	the quotient of 51 and 17
Translate.	$51 \div 17$
Divide.	3

We could just as correctly have translated *the quotient of* 51 *and* 17 using the notation

$17\overline{)51}$ or $\frac{51}{17}$.

Note:
Exercise:

Problem: Translate and simplify: the quotient of 91 and 13.

Solution:

$91 \div 13$; 7

Note:
Exercise:

Problem: Translate and simplify: the quotient of 52 and 13.

Solution:

52 ÷ 13; 4

Divide Whole Numbers in Applications

We will use the same strategy we used in previous sections to solve applications. First, we determine what we are looking for. Then we write a phrase that gives the information to find it. We then translate the phrase into math notation and simplify it to get the answer. Finally, we write a sentence to answer the question.

Example:
Exercise:

Problem:

Cecelia bought a 160-ounce box of oatmeal at the big box store. She wants to divide the 160 ounces of oatmeal into 8-ounce servings. She will put each serving into a plastic bag so she can take one bag to work each day. How many servings will she get from the big box?

Solution:
Solution

We are asked to find the how many servings she will get from the big box.

Write a phrase.	160 ounces divided by 8 ounces
Translate to math notation.	$160 \div 8$
Simplify by dividing.	20
Write a sentence to answer the question.	Cecelia will get 20 servings from the big box.

Note:
Exercise:

Problem:

Marcus is setting out animal crackers for snacks at the preschool. He wants to put 9 crackers in each cup. One box of animal crackers contains 135 crackers. How many cups can he fill from one box of crackers?

Solution:

Marcus can fill 15 cups.

Note:
Exercise:

Problem:

Andrea is making bows for the girls in her dance class to wear at the recital. Each bow takes 4 feet of ribbon, and 36 feet of ribbon are on one spool. How many bows can Andrea make from one spool of ribbon?

Solution:

Andrea can make 9 bows.

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Dividing Whole Numbers
- Dividing Whole Numbers No Remainder
- Dividing Whole Numbers With Remainder

Key Concepts

Operation	Notation	Expression	Read as	Result
Division	$\div$ $\frac{a}{b}$ $b\overline{)a}$ a/b	$12 \div 4$ $\frac{12}{4}$ $4\overline{)12}$ $12/4$	Twelve divided by four	the quotient of 12 and 4

- **Division Properties of One**
  - Any number (except 0) divided by itself is one. $a \div a = 1$
  - Any number divided by one is the same number. $a \div 1 = a$
- **Division Properties of Zero**
  - Zero divided by any number is 0. $0 \div a = 0$
  - Dividing a number by zero is undefined. $a \div 0$ undefined
- **Divide whole numbers.**

Divide the first digit of the	If the divisor is larger than the first digit of the dividend, divide the first

dividend by the divisor. two digits of the dividend by the divisor, and so on.
Write the quotient above the dividend.
Multiply the quotient by the divisor and write the product under the dividend.
Subtract that product from the dividend.
Bring down the next digit of the dividend.
Repeat from Step 1 until there are no more digits in the dividend to bring down.
Check by multiplying the quotient times the divisor.

Section Exercises

Practice Makes Perfect

Use Division Notation

In the following exercises, translate from math notation to words.

Exercise:

Problem: $54 \div 9$

Solution:

fifty-four divided by nine; the quotient of fifty-four and nine

Exercise:

Problem: $\frac{56}{7}$

Exercise:

Problem: $\frac{32}{8}$

Solution:

thirty-two divided by eight; the quotient of thirty-two and eight

Exercise:

Problem: $6\overline{)42}$

Exercise:

Problem: $48 \div 6$

Solution:

forty-eight divided by six; the quotient of forty-eight and six

Exercise:

Problem: $\frac{63}{9}$

Exercise:

Problem: $7\overline{)63}$

Solution:

sixty-three divided by seven; the quotient of sixty-three and seven

Exercise:

Problem: $72 \div 8$

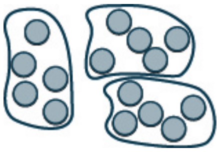
Model Division of Whole Numbers

In the following exercises, model the division.

Exercise:

Problem: $15 \div 5$

Solution:



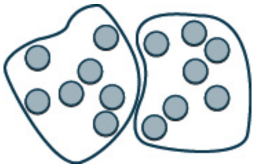
Exercise:

Problem: $10 \div 5$

Exercise:

Problem: $\frac{14}{7}$

Solution:



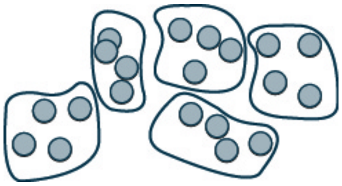
Exercise:

Problem: $\frac{18}{6}$

Exercise:

Problem: $4\overline{)20}$

Solution:



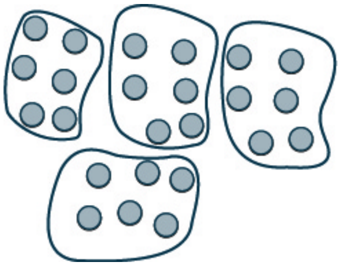
Exercise:

Problem: $3\overline{)15}$

Exercise:

Problem: $24 \div 6$

Solution:



Exercise:

Problem: $16 \div 4$

Divide Whole Numbers

In the following exercises, divide. Then check by multiplying.

Exercise:

Problem: $18 \div 2$

Solution:

9

Exercise:

Problem: $14 \div 2$

Exercise:

Problem: $\frac{27}{3}$

Solution:

9

Exercise:

Problem: $\frac{30}{3}$

Exercise:

Problem: $4\overline{)28}$

Solution:

7

Exercise:

Problem: $4\overline{)36}$

Exercise:

Problem: $\frac{45}{5}$

Solution:

9

Exercise:

Problem: $\frac{35}{5}$

Exercise:

Problem: $72/8$

Solution:

9

Exercise:

Problem: $8\overline{)64}$

Exercise:

Problem: $\frac{35}{7}$

Solution:

5

Exercise:

Problem: $42 \div 7$

Exercise:

Problem: $15\overline{)15}$

Solution:

1

Exercise:

Problem: $12\overline{)12}$

Exercise:

Problem: $43 \div 43$

Solution:

1

Exercise:

Problem: $37 \div 37$

Exercise:

Problem: $\frac{23}{1}$

Solution:

23

Exercise:

Problem: $\frac{29}{1}$

Exercise:

Problem: $19 \div 1$

Solution:

19

Exercise:

Problem: $17 \div 1$

Exercise:

Problem: $0 \div 4$

Solution:

0

Exercise:

Problem: $0 \div 8$

Exercise:

Problem: $\frac{5}{0}$

Solution:

undefined

Exercise:

Problem: $\frac{9}{0}$

Exercise:

Problem: $\frac{26}{0}$

Solution:

undefined

Exercise:

Problem: $\frac{32}{0}$

Exercise:

Problem: $12\overline{)0}$

Solution:

0

Exercise:

Problem: $16\overline{)0}$

Exercise:

Problem: $72 \div 3$

Solution:

24

Exercise:

Problem: $57 \div 3$

Exercise:

Problem: $\frac{96}{8}$

Solution:

12

Exercise:

Problem: $\frac{78}{6}$

Exercise:

Problem: $5\overline{)465}$

Solution:

93

Exercise:

Problem: $4\overline{)528}$

Exercise:

Problem: $924 \div 7$

Solution:

132

Exercise:

Problem: $861 \div 7$

Exercise:

Problem: $\frac{5,226}{6}$

Solution:

871

Exercise:

Problem: $\frac{3,776}{8}$

Exercise:

Problem: $4\overline{)31,324}$

Solution:

7,831

Exercise:

Problem: $5\overline{)46,855}$

Exercise:

Problem: $7{,}209 \div 3$

Solution:

2,403

Exercise:

Problem: $4{,}806 \div 3$

Exercise:

Problem: $5{,}406 \div 6$

Solution:

901

Exercise:

Problem: $3{,}208 \div 4$

Exercise:

Problem: $4\overline{)2{,}816}$

Solution:

704

Exercise:

Problem: $6\overline{)3{,}624}$

Exercise:

Problem: $\frac{91{,}881}{9}$

Solution:

10,209

Exercise:

Problem: $\frac{83{,}256}{8}$

Exercise:

Problem: $2{,}470 \div 7$

Solution:

352 R6

Exercise:

Problem: $3{,}741 \div 7$

Exercise:

Problem: $8\overline{)55{,}305}$

Solution:

6,913 R1

Exercise:

Problem: $9\overline{)51{,}492}$

Exercise:

Problem: $\frac{431{,}174}{5}$

Solution:

86,234 R4

Exercise:

Problem: $\frac{297{,}277}{4}$

Exercise:

Problem: $130{,}016 \div 3$

Solution:

43,338 R2

Exercise:

Problem: $105{,}609 \div 2$

Exercise:

Problem: $15\overline{)5{,}735}$

Solution:

382 R5

Exercise:

Problem: $\frac{4{,}933}{21}$

Exercise:

Problem: $56{,}883 \div 67$

Solution:

849

Exercise:

Problem: $43{,}725/75$

Exercise:

Problem: $\frac{30{,}144}{314}$

Solution:

96

Exercise:

Problem: $26{,}145 \div 415$

Exercise:

Problem: $273\overline{)542{,}195}$

Solution:

1,986 R17

Exercise:

Problem: $816{,}243 \div 462$

Mixed Practice

In the following exercises, simplify.

Exercise:

Problem: $15\,(204)$

Solution:

3,060

Exercise:

Problem: $74 \cdot 391$

Exercise:

Problem: $256 - 184$

Solution:

72

Exercise:

Problem: $305 - 262$

Exercise:

Problem: $719 + 341$

Solution:

1,060

Exercise:

Problem: $647 + 528$

Exercise:

Problem: $25\overline{)875}$

Solution:

35

Exercise:

Problem: $1104 \div 23$

Translate Word Phrases to Algebraic Expressions

In the following exercises, translate and simplify.

Exercise:

Problem: the quotient of 45 and 15

Solution:

45 ÷ 15; 3

Exercise:

Problem: the quotient of 64 and 16

Exercise:

Problem: the quotient of 288 and 24

Solution:

288 ÷ 24; 12

Exercise:

Problem: the quotient of 256 and 32

Divide Whole Numbers in Applications

In the following exercises, solve.

Exercise:

Problem:

Trail mix Ric bought 64 ounces of trail mix. He wants to divide it into small bags, with 2 ounces of trail mix in each bag. How many bags can Ric fill?

Solution:

Ric can fill 32 bags.

Exercise:

Problem:

Crackers Evie bought a 42 ounce box of crackers. She wants to divide it into bags with 3 ounces of crackers in each bag. How many bags can Evie fill?

Exercise:

Problem:

Astronomy class There are 125 students in an astronomy class. The professor assigns them into groups of 5. How many groups of students are there?

Solution:

There are 25 groups.

Exercise:

Problem:

Flower shop Melissa’s flower shop got a shipment of 152 roses. She wants to make bouquets of 8 roses each. How many bouquets can Melissa make?

Exercise:

Problem:

Baking One roll of plastic wrap is 48 feet long. Marta uses 3 feet of plastic wrap to wrap each cake she bakes. How many cakes can she wrap from one roll?

Solution:

Marta can wrap 16 cakes from 1 roll.

Exercise:

Problem:

Dental floss One package of dental floss is 54 feet long. Brian uses 2 feet of dental floss every day. How many days will one package of dental floss last Brian?

Mixed Practice

In the following exercises, solve.

Exercise:

Problem:

Miles per gallon Susana’s hybrid car gets 45 miles per gallon. Her son’s truck gets 17 miles per gallon. What is the difference in miles per gallon between Susana’s car and her son’s truck?

Solution:

The difference is 28 miles per gallon.

Exercise:

Problem:

Distance Mayra lives 53 miles from her mother’s house and 71 miles from her mother-in-law’s house. How much farther is Mayra from her mother-in-law’s house than from her mother’s house?

Exercise:

Problem:

Field trip The 45 students in a Geology class will go on a field trip, using the college’s vans. Each van can hold 9 students. How many vans will they need for the field trip?

Solution:

They will need 5 vans for the field trip

Exercise:

Problem:

Potting soil Aki bought a 128 ounce bag of potting soil. How many 4 ounce pots can he fill from the bag?

Exercise:

Problem:

Hiking Bill hiked 8 miles on the first day of his backpacking trip, 14 miles the second day, 11 miles the third day, and 17 miles the fourth day. What is the total number of miles Bill hiked?

Solution:

Bill hiked 50 miles

Exercise:

Problem:

Reading Last night Emily read 6 pages in her Business textbook, 26 pages in her History text, 15 pages in her Psychology text, and 9 pages in her math text. What is the total number of pages Emily read?

Exercise:

Problem:

Patients LaVonne treats 12 patients each day in her dental office. Last week she worked 4 days. How many patients did she treat last week?

Solution:

LaVonne treated 48 patients last week.

Exercise:

Problem:

Scouts There are 14 boys in Dave’s scout troop. At summer camp, each boy earned 5 merit badges. What was the total number of merit badges earned by Dave’s scout troop at summer camp?

Writing Exercises

Exercise:

Problem: Explain how you use the multiplication facts to help with division.

Solution:

Answers may vary. Using multiplication facts can help you check your answers once you’ve finished division.

Exercise:

Problem:

Oswaldo divided 300 by 8 and said his answer was 37 with a remainder of 4. How can you check to make sure he is correct?

Everyday Math

Exercise:

Problem:

Contact lenses Jenna puts in a new pair of contact lenses every 14 days. How many pairs of contact lenses does she need for 365 days?

Solution:

Jenna uses 26 pairs of contact lenses, but there is 1 day left over, so she needs 27 pairs for 365 days.

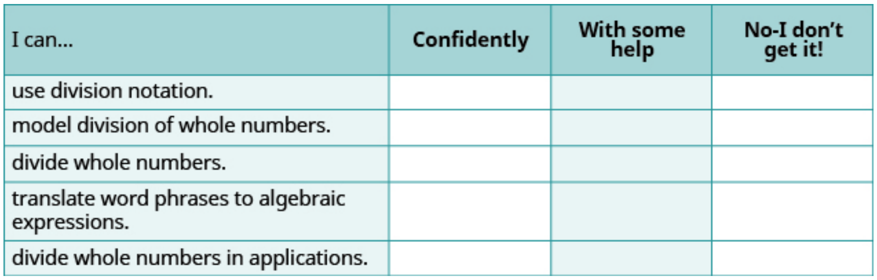
Exercise:

Problem:

Cat food One bag of cat food feeds Lara’s cat for 25 days. How many bags of cat food does Lara need for 365 days?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
use division notation.			
model division of whole numbers.			
divide whole numbers.			
translate word phrases to algebraic expressions.			
divide whole numbers in applications.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

Chapter Review Exercises

Introduction to Whole Numbers

Identify Counting Numbers and Whole Numbers

In the following exercises, determine which of the following are (a) counting numbers (b) whole numbers.

Exercise:

Problem: $0, 2, 99$

Solution:

ⓐ 2, 99
ⓑ 0, 2, 99

Exercise:

Problem: $0, 3, 25$

Exercise:

Problem: $0, 4, 90$

Solution:

ⓐ 4, 90
ⓑ 0, 4, 90

Exercise:

Problem: $0, 1, 75$

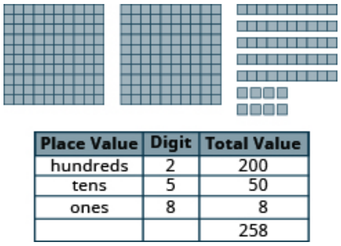
Model Whole Numbers

In the following exercises, model each number using base-10 blocks and then show its value using place value notation.

Exercise:

Problem: 258

Solution:



Place Value	Digit	Total Value
hundreds	2	200
tens	5	50
ones	8	8
		258

Exercise:

Problem: 104

Identify the Place Value of a Digit

In the following exercises, find the place value of the given digits.

Exercise:

Problem: 472,981

ⓐ 8
ⓑ 4
ⓒ 1
ⓓ 7
ⓔ 2

Solution:

ⓐ tens
ⓑ hundred thousands
ⓒ ones
ⓓ thousands
ⓔ ten thousands

Exercise:

Problem: 12,403,295

ⓐ 4
ⓑ 0
ⓒ 1
ⓓ 9
ⓔ 3

Use Place Value to Name Whole Numbers

In the following exercises, name each number in words.

Exercise:

Problem: 5,280

Solution:

Five thousand two hundred eighty

Exercise:

Problem: 204,614

Exercise:

Problem: 5,012,582

Solution:

Five million twelve thousand five hundred eighty-two

Exercise:

Problem: 31,640,976

Use Place Value to Write Whole Numbers

In the following exercises, write as a whole number using digits.

Exercise:

Problem: six hundred two

Exercise:

Problem: fifteen thousand, two hundred fifty-three

Solution:

15,253

Exercise:

Problem: three hundred forty million, nine hundred twelve thousand, sixty-one

---

Solution:

340,912,061

Exercise:

Problem: two billion, four hundred ninety-two million, seven hundred eleven thousand, two

Round Whole Numbers

In the following exercises, round to the nearest ten.

Exercise:

Problem: 412

Solution:

410

Exercise:

Problem: 648

Exercise:

Problem: 3,556

Solution:

3,560

Exercise:

Problem: 2,734

In the following exercises, round to the nearest hundred.

Exercise:

Problem: 38,975

Solution:

39,000

Exercise:

Problem: 26,849

Exercise:

Problem: 81,486

Solution:

81,500

Exercise:

Problem: 75,992

Add Whole Numbers

Use Addition Notation

In the following exercises, translate the following from math notation to words.

Exercise:

Problem: $4+3$

Solution:

four plus three; the sum of four and three

Exercise:

Problem: $25+18$

Exercise:

Problem: $571+629$

Solution:

five hundred seventy-one plus six hundred twenty-nine; the sum of five hundred seventy-one and six hundred twenty-nine

Exercise:

Problem: $10{,}085+3{,}492$

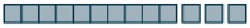
Model Addition of Whole Numbers

In the following exercises, model the addition.

Exercise:

Problem: $6+7$

Solution:



Exercise:

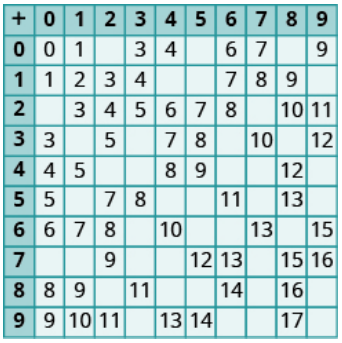
Problem: $38+14$

Add Whole Numbers

In the following exercises, fill in the missing values in each chart.

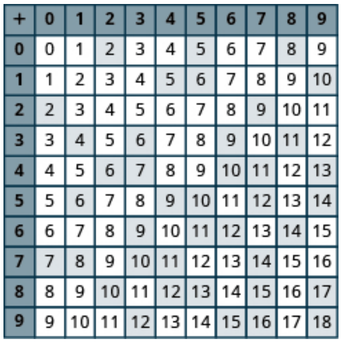
Exercise:

Problem:



+	0	1	2	3	4	5	6	7	8	9
0	0	1		3	4		6	7		9
1	1	2	3	4			7	8	9	
2		3	4	5	6	7	8		10	11
3	3		5		7	8		10		12
4	4	5			8	9			12	
5	5		7	8			11		13	
6	6	7	8		10			13		15
7			9			12	13		15	16
8	8	9		11			14		16	
9	9	10	11		13	14			17	

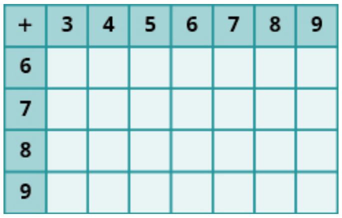
Solution:



+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Exercise:

Problem:



+	3	4	5	6	7	8	9
6							
7							
8							
9							

In the following exercises, add.

Exercise:

Problem: ⓐ $0 + 19$ ⓑ $19 + 0$

Solution:

ⓐ 19
ⓑ 19

Exercise:

Problem: ⓐ $0 + 480$ ⓑ $480 + 0$

Exercise:

Problem: ⓐ $7+6$ ⓑ $6+7$

Solution:

ⓐ 13
ⓑ 13

Exercise:

Problem: ⓐ $23+18$ ⓑ $18+23$

Exercise:

Problem: $44+35$

Solution:

82

Exercise:

Problem: $63+29$

Exercise:

Problem: $96+58$

Solution:

154

Exercise:

Problem: $375+591$

Exercise:

Problem: $7{,}281+12{,}546$

Solution:

19,827

Exercise:

Problem: $5{,}280+16{,}324+9{,}731$

Translate Word Phrases to Math Notation

In the following exercises, translate each phrase into math notation and then simplify.

Exercise:

Problem: the sum of 30 and 12

Solution:

30 + 12; 42

Exercise:

Problem: 11 increased by 8

Exercise:

Problem: 25 more than 39

Solution:

39 + 25; 64

Exercise:

Problem: total of 15 and 50

Add Whole Numbers in Applications

In the following exercises, solve.

Exercise:

Problem:

Shopping for an interview Nathan bought a new shirt, tie, and slacks to wear to a job interview. The shirt cost \$24, the tie cost \$14, and the slacks cost \$38. What was Nathan’s total cost?

Solution:

\$76

Exercise:

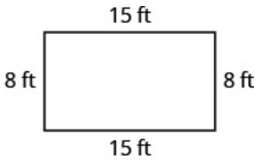
Problem:

Running Jackson ran 4 miles on Monday, 12 miles on Tuesday, 1 mile on Wednesday, 8 miles on Thursday, and 5 miles on Friday. What was the total number of miles Jackson ran?

In the following exercises, find the perimeter of each figure.

Exercise:

Problem:

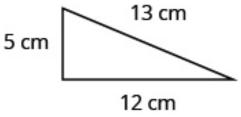


Solution:

46 feet

Exercise:

Problem:



Subtract Whole Numbers

Use Subtraction Notation

In the following exercises, translate the following from math notation to words.

Exercise:

Problem: $14-5$

Solution:

fourteen minus five; the difference of fourteen and five

Exercise:

Problem: $40-15$

Exercise:

Problem: $351-249$

Solution:

three hundred fifty-one minus two hundred forty-nine; the difference between three hundred fifty-one and two hundred forty-nine

Exercise:

Problem: $5{,}724-2{,}918$

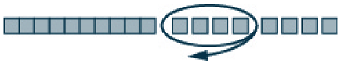
Model Subtraction of Whole Numbers

In the following exercises, model the subtraction.

Exercise:

Problem: $18-4$

Solution:



Exercise:

Problem: $41 - 29$

Subtract Whole Numbers

In the following exercises, subtract and then check by adding.

Exercise:

Problem: $8 - 5$

Solution:

3

Exercise:

Problem: $12 - 7$

Exercise:

Problem: $23 - 9$

Solution:

14

Exercise:

Problem: $46 - 21$

Exercise:

Problem: $82 - 59$

Solution:

23

Exercise:

Problem: $110 - 87$

Exercise:

Problem: $539 - 217$

Solution:

322

Exercise:

Problem: $415 - 296$

Exercise:

Problem: $1{,}020 - 640$

Solution:

380

Exercise:

Problem: $8{,}355 - 3{,}947$

Exercise:

Problem: $10{,}000 - 15$

Solution:

9,985

Exercise:

Problem: $54{,}925 - 35{,}647$

Translate Word Phrases to Math Notation

In the following exercises, translate and simplify.

Exercise:

Problem: the difference of nineteen and thirteen

Solution:

$19 - 13$; 6

Exercise:

Problem: subtract sixty-five from one hundred

Exercise:

Problem: seventy-four decreased by eight

Solution:

$74 - 8$; 66

Exercise:

Problem: twenty-three less than forty-one

Subtract Whole Numbers in Applications

In the following exercises, solve.

Exercise:

Problem:

Temperature The high temperature in Peoria one day was 86 degrees Fahrenheit and the low temperature was 28 degrees Fahrenheit. What was the difference between the high and low temperatures?

Solution:

58 degrees Fahrenheit

Exercise:

Problem:

Savings Lynn wants to go on a cruise that costs $\$2,485$. She has $\$948$ in her vacation savings account. How much more does she need to save in order to pay for the cruise?

Multiply Whole Numbers

Use Multiplication Notation

In the following exercises, translate from math notation to words.

Exercise:

Problem: 8×5

Solution:

eight times five the product of eight and five

Exercise:

Problem: $6 \cdot 14$

Exercise:

Problem: $(10)(95)$

Solution:

ten times ninety-five; the product of ten and ninety-five

Exercise:

Problem: $54(72)$

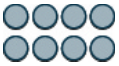
Model Multiplication of Whole Numbers

In the following exercises, model the multiplication.

Exercise:

Problem: 2×4

Solution:



Exercise:

Problem: 3×8

Multiply Whole Numbers

In the following exercises, fill in the missing values in each chart.

Exercise:

Problem:

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0		0		0	0
1	0	1	2		4	5	6	7		9
2	0		4		8	10		14	16	
3		3		9			18		24	
4	0	4		12			24			36
5	0	5	10		20		30	35	40	45
6			12	18			36	42		54
7	0	7		21		35			56	63
8	0	8	16		32		48		64	
9			18	27	36			63	72	

Solution:

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

Exercise:

Problem:

×	3	4	5	6	7	8	9
6							
7							
8							
9							

In the following exercises, multiply.

Exercise:

Problem: $0 \cdot 14$

Solution:

0

Exercise:

Problem: $(256)0$

Exercise:

Problem: $1 \cdot 99$

Solution:

99

Exercise:

Problem: $(4{,}789)1$

Exercise:

Problem: ⓐ $7 \cdot 4$ ⓑ $4 \cdot 7$

Solution:

ⓐ 28
ⓑ 28

Exercise:

Problem: $(25)(6)$

Exercise:

Problem: $9{,}261 \times 3$

Solution:

27,783

Exercise:

Problem: $48 \cdot 76$

Exercise:

Problem: $64 \cdot 10$

Solution:

640

Exercise:

Problem: $1,000(22)$

Exercise:

Problem: 162×493

Solution:

79,866

Exercise:

Problem: $(601)(943)$

Exercise:

Problem: $3,624 \times 517$

Solution:

1,873,608

Exercise:

Problem: $10,538 \cdot 22$

Translate Word Phrases to Math Notation

In the following exercises, translate and simplify.

Exercise:

Problem: the product of 15 and 28

Solution:

15(28); 420

Exercise:

Problem: ninety-four times thirty-three

Exercise:

Problem: twice 575

Solution:

2(575); 1,150

Exercise:

Problem: ten times two hundred sixty-four

Multiply Whole Numbers in Applications

In the following exercises, solve.

Exercise:

Problem:

Gardening Geniece bought 8 packs of marigolds to plant in her yard. Each pack has 6 flowers. How many marigolds did Geniece buy?

Solution:

48 marigolds

Exercise:

Problem:

Cooking Ratika is making rice for a dinner party. The number of cups of water is twice the number of cups of rice. If Ratika plans to use 4 cups of rice, how many cups of water does she need?

Exercise:

Problem:

Multiplex There are twelve theaters at the multiplex and each theater has 150 seats. What is the total number of seats at the multiplex?

Solution:

1,800 seats

Exercise:

Problem:

Roofing Lewis needs to put new shingles on his roof. The roof is a rectangle, 30 feet by 24 feet. What is the area of the roof?

Divide Whole Numbers

Use Division Notation

Translate from math notation to words.

Exercise:

Problem: $54 \div 9$

Solution:

fifty-four divided by nine; the quotient of fifty-four and nine

Exercise:

Problem: $42/7$

Exercise:

Problem: $\frac{72}{8}$

Solution:

seventy-two divided by eight; the quotient of seventy-two and eight

Exercise:

Problem: $6\overline{)48}$

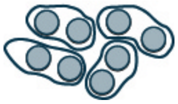
Model Division of Whole Numbers

In the following exercises, model.

Exercise:

Problem: $8 \div 2$

Solution:



Exercise:

Problem: $3\overline{)12}$

Divide Whole Numbers

In the following exercises, divide. Then check by multiplying.

Exercise:

Problem: $14 \div 2$

Solution:

7

Exercise:

Problem: $\frac{32}{8}$

Exercise:

Problem: $52 \div 4$

Solution:

13

Exercise:

Problem: $26\overline{)26}$

Exercise:

Problem: $\frac{97}{1}$

Solution:

97

Exercise:

Problem: $0 \div 52$

Exercise:

Problem: $100 \div 0$

Solution:

undefined

Exercise:

Problem: $\frac{355}{5}$

Exercise:

Problem: $3828 \div 6$

Solution:

638

Exercise:

Problem: $31\overline{)1{,}519}$

Exercise:

Problem: $\frac{7505}{25}$

Solution:

300 R5

Exercise:

Problem: $5{,}166 \div 42$

Translate Word Phrases to Math Notation

In the following exercises, translate and simplify.

Exercise:

Problem: the quotient of 64 and 16

Solution:

64 ÷ 16; 4

Exercise:

Problem: the quotient of 572 and 52

Divide Whole Numbers in Applications

In the following exercises, solve.

Exercise:

Problem:

Ribbon One spool of ribbon is 27 feet. Lizbeth uses 3 feet of ribbon for each gift basket that she wraps. How many gift baskets can Lizbeth wrap from one spool of ribbon?

Solution:

9 baskets

Exercise:

Problem:

Juice One carton of fruit juice is 128 ounces. How many 4 ounce cups can Shayla fill from one carton of juice?

Chapter Practice Test

Exercise:

Problem: Determine which of the following numbers are

ⓐ counting numbers
ⓑ whole numbers.

$0, 4, 87$

Solution:

ⓐ 4, 87
ⓑ 0, 4, 8

Exercise:

Problem: Find the place value of the given digits in the number 549,362.

ⓐ 9
ⓑ 6
ⓒ 2
ⓓ 5

Exercise:

Problem: Write each number as a whole number using digits.

ⓐ six hundred thirteen
ⓑ fifty-five thousand two hundred eight

Solution:

ⓐ 613
ⓑ 55,208

Exercise:

Problem: Round $25{,}849$ to the nearest hundred.

Simplify.

Exercise:

Problem: $45 + 23$

Solution:

68

Exercise:

Problem: $65 - 42$

Exercise:

Problem: $85 \div 5$

Solution:

17

Exercise:

Problem: $1{,}000 \ \times \ 8$

Exercise:

Problem: $90 - 58$

Solution:

32

Exercise:

Problem: $73+89$

Exercise:

Problem: $(0)(12{,}675)$

Solution:

0

Exercise:

Problem: $634+255$

Exercise:

Problem: $\frac{0}{9}$

Solution:

0

Exercise:

Problem: $8\overline{)128}$

Exercise:

Problem: $145-79$

Solution:

66

Exercise:

Problem: $299+836$

Exercise:

Problem: $7\cdot 475$

Solution:

$3{,}325$

Exercise:

Problem: $8{,}528+704$

Exercise:

Problem: $35(14)$

Solution:

490

Exercise:

Problem: $\frac{26}{0}$

Exercise:

Problem: $733 - 291$

Solution:

442

Exercise:

Problem: $4{,}916 - 1{,}538$

Exercise:

Problem: $495 \div 45$

Solution:

11

Exercise:

Problem: $52 \ \times \ 983$

Translate each phrase to math notation and then simplify.

Exercise:

Problem: The sum of 16 and 58

Solution:

16 + 58; 74

Exercise:

Problem: The product of 9 and 15

Exercise:

Problem: The difference of 32 and 18

Solution:

32 − 18; 14

Exercise:

Problem: The quotient of 63 and 21

Exercise:

Problem: Twice 524

Solution:

2(524); 1,048

Exercise:

Problem: 29 more than 32

Exercise:

Problem: 50 less than 300

Solution:

300 − 50; 250

In the following exercises, solve.

Exercise:

Problem:

LaVelle buys a jumbo bag of 84 candies to make favor bags for her son’s party. If she wants to make 12 bags, how many candies should she put in each bag?

Exercise:

Problem:

Last month, Stan’s take-home pay was $3,816 and his expenses were $3,472. How much of his take-home pay did Stan have left after he paid his expenses?

Solution:

Stan had $344 left.

Exercise:

Problem:

Each class at Greenville School has 22 children enrolled. The school has 24 classes. How many children are enrolled at Greenville School?

Exercise:

Problem:

Clayton walked 12 blocks to his mother’s house, 6 blocks to the gym, and 9 blocks to the grocery store before walking the last 3 blocks home. What was the total number of blocks that Clayton walked?

Solution:

Clayton walked 30 blocks.

Glossary

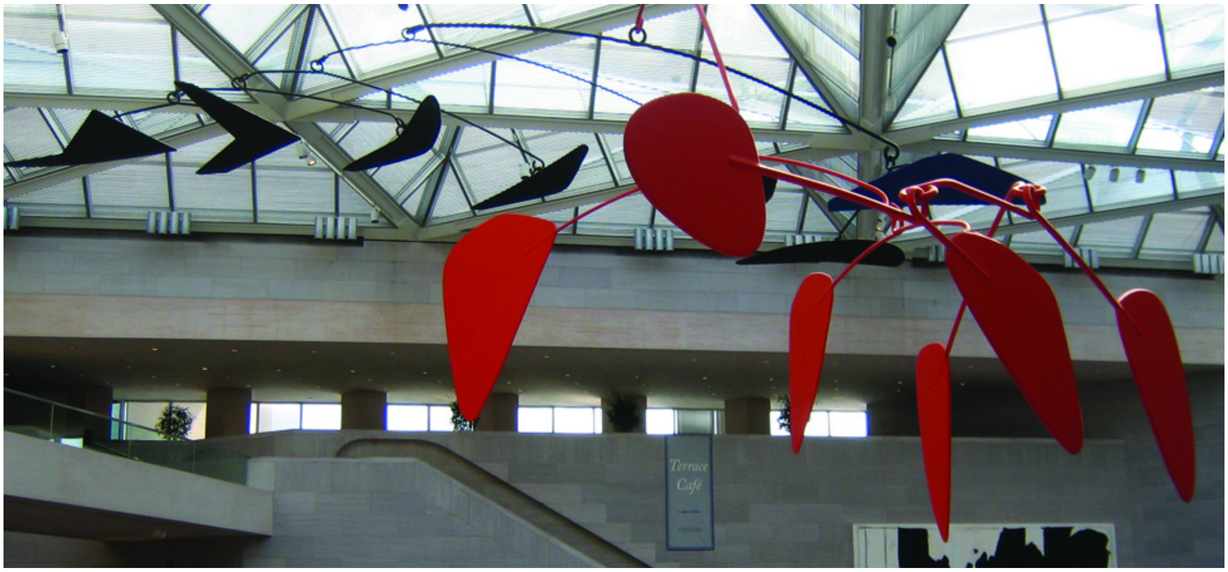
dividend
When dividing two numbers, the dividend is the number being divided.

divisor
When dividing two numbers, the divisor is the number dividing the dividend.

quotient
The quotient is the result of dividing two numbers.

# Introduction to Solving Linear Equations
class="introduction"

A Calder mobile is balanced and has several elements on each side. (credit: paurian, Flickr)



Teetering high above the floor, this amazing mobile remains aloft thanks to its carefully balanced mass. Any shift in either direction could cause the mobile to become lopsided, or even crash downward. In this chapter, we will solve equations by keeping quantities on both sides of an equal sign in perfect balance.

Solve Equations Using the Subtraction and Addition Properties of Equality
By the end of this section, you will be able to:

- Solve equations using the Subtraction and Addition Properties of Equality
- Solve equations that need to be simplified
- Translate an equation and solve
- Translate and solve applications

Note:
Before you get started, take this readiness quiz.

1. Solve: $n-12=16$.
   If you missed this problem, review [link].
2. Translate into algebra ‘five less than x.’
   If you missed this problem, review [link].
3. Is $x=2$ a solution to $5x-3=7$?
   If you missed this problem, review [link].

We are now ready to “get to the good stuff.” You have the basics down and are ready to begin one of the most important topics in algebra: solving equations. The applications are limitless and extend to all careers and fields. Also, the skills and techniques you learn here will help improve your critical thinking and problem-solving skills. This is a great benefit of studying mathematics and will be useful in your life in ways you may not see right now.

Solve Equations Using the Subtraction and Addition Properties of Equality

We began our work solving equations in previous chapters. It has been a while since we have seen an equation, so we will review some of the key concepts before we go any further.

We said that solving an equation is like discovering the answer to a puzzle. The purpose in solving an equation is to find the value or values of the variable that make each side of the equation the same. Any value of the variable that makes the equation true is called a solution to the equation. It is the answer to the puzzle.

Note:
Solution of an Equation
A **solution of an equation** is a value of a variable that makes a true statement when substituted into the equation.

In the earlier sections, we listed the steps to determine if a value is a solution. We restate them here.

Note:
Determine whether a number is a solution to an equation.

Substitute the number for the variable in the equation.
Simplify the expressions on both sides of the equation.
Determine whether the resulting equation is true.

- If it is true, the number is a solution.
- If it is not true, the number is not a solution.

Example:
Exercise:

Problem: Determine whether $y = \frac{3}{4}$ is a solution for $4y + 3 = 8y$.

Solution:
Solution

	$4y + 3 = 8y$
Substitute $\frac{3}{4}$ for y.	$4\left(\frac{3}{4}\right) + 3 \overset{?}{=} 8\left(\frac{3}{4}\right)$
Multiply.	$3 + 3 \overset{?}{=} 6$
Add.	$6 = 6$ ✓

Since $y = \frac{3}{4}$ results in a true equation, $\frac{3}{4}$ is a solution to the equation $4y + 3 = 8y$.

Note:
Exercise:

Problem: Is $y = \frac{2}{3}$ a solution for $9y + 2 = 6y$?

Solution:

no

Note:
Exercise:

Problem: Is $y = \frac{2}{5}$ a solution for $5y - 3 = 10y$?

Solution:

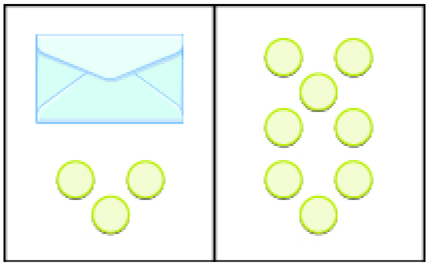
no

We introduced the Subtraction and Addition Properties of Equality in Solving Equations Using the Subtraction and Addition Properties of Equality. In that section, we modeled how these properties work and then applied them to solving equations with whole numbers. We used these properties again each time we introduced a new system of numbers. Let’s review those properties here.

Note:
Subtraction and Addition Properties of Equality
Subtraction Property of Equality
For all real numbers a, b, and c, if $a = b$, then $a - c = b - c$.
Addition Property of Equality
For all real numbers a, b, and c, if $a = b$, then $a + c = b + c$.

When you add or subtract the same quantity from both sides of an equation, you still have equality.

We introduced the Subtraction Property of Equality earlier by modeling equations with envelopes and counters. [link] models the equation $x+3=8$.



The goal is to isolate the variable on one side of the equation. So we ‘took away’ 3 from both sides of the equation and found the solution $x=5$.

Some people picture a balance scale, as in [link], when they solve equations.

1 mass on each side = balanced

2 masses on each side = balanced

1 mass on one side and 2 masses on the other = unbalanced

The quantities on both sides of the equal sign in an equation are equal, or balanced. Just as with the balance scale, whatever you do to one side of the equation you must also do to the other to keep it balanced.

Let’s review how to use Subtraction and Addition Properties of Equality to solve equations. We need to isolate the variable on one side of the equation. And we check our solutions by substituting the value into the equation to make sure we have a true statement.

Example:
Exercise:

Problem: Solve: $x + 11 = -3$.

Solution:
Solution

To isolate x, we undo the addition of 11 by using the Subtraction Property of Equality.

		$x - 11 = -3$
Subtract 11 from each side to "undo" the addition.		$x + 11 - 11 = -3 - 11$
Simplify.		$x = -14$
Check:	$x - 11 = -3$	
Substitute $x = -14$.	$-14 + 11 \overset{?}{=} -3$	
	$-3 = -3$ ✓	

Since $x = -14$ makes $x + 11 = -3$ a true statement, we know that it is a solution to the equation.

Note:
Exercise:

Problem: Solve: $x + 9 = -7.$

Solution:

$x = -16$

Note:
Exercise:

Problem: Solve: $x + 16 = -4.$

Solution:

$x = -20$

In the original equation in the previous example, 11 was added to the x, so we subtracted 11 to ‘undo’ the addition. In the next example, we will need to ‘undo’ subtraction by using the Addition Property of Equality.

Example:
Exercise:

Problem: Solve: $m - 4 = -5.$

Solution:
Solution

		$m+4=-5$
Add 4 to each side to "undo" the subtraction.		$m+4-4=-5-4$
Simplify.		$m=-1$
Check:	$m+4=-5$	
Substitute $m=-1$.	$-1+4\stackrel{?}{=}-5$	
	$-5=-5$ ✓	
		The solution to $m-4=-5$ is $m=-1$.

Note:

Exercise:

Problem: Solve: $n - 6 = -7$.

Solution:

−1

Note:
Exercise:

Problem: Solve: $x - 5 = -9$.

Solution:

−4

Now let’s review solving equations with fractions.

Example:
Exercise:

Problem: Solve: $n - \frac{3}{8} = \frac{1}{2}$.

Solution:
Solution

		$n-\frac{3}{8}=\frac{1}{2}$
Use the Addition Property of Equality.		$n-\frac{3}{8}+\frac{3}{8}=\frac{1}{2}+\frac{3}{8}$
Find the LCD to add the fractions on the right.		$n-\frac{3}{8}+\frac{3}{8}=\frac{4}{8}+\frac{3}{8}$
Simplify		$n=\frac{7}{8}$
Check:	$n-\frac{3}{8}=\frac{1}{2}$	
Substitute $n=\frac{7}{8}$.	$\frac{7}{8}-\frac{3}{8}\overset{?}{=}\frac{1}{2}$	
Subtract.	$\frac{4}{8}\overset{?}{=}\frac{1}{2}$	
Simplify.	$\frac{1}{2}=\frac{1}{2}$ ✓	
The solution checks.		

Note:
Exercise:

Problem: Solve: $p-\frac{1}{3}=\frac{5}{6}$.

Solution:

$p=\frac{7}{6}$

Note:
Exercise:

Problem: Solve: $q-\frac{1}{2}=\frac{1}{6}$.

Solution:

$q=\frac{2}{3}$

In Solve Equations with Decimals, we solved equations that contained decimals. We’ll review this next.

Example:
Exercise:

Problem: Solve $a-3.7=4.3$.

Solution:
Solution

		$a - 3.7 = 4.3$
Use the Addition Property of Equality.		$a - 3.7 + 3.7 = 4.3 + 3.7$
Add.		$a = 8$
Check:	$a - 3.7 = 4.3$	
Substitute $a = 8$.	$8 - 3.7 \overset{?}{=} 4.3$	
Simplify.	$4.3 = 4.3$ ✓	
The solution checks.		

Note:

Exercise:

Problem: Solve: $b - 2.8 = 3.6$.

Solution:

b = 6.4

Note:
Exercise:

Problem: Solve: $c - 6.9 = 7.1$.

Solution:

c = 14

Solve Equations That Need to Be Simplified

In the examples up to this point, we have been able to isolate the variable with just one operation. Many of the equations we encounter in algebra will take more steps to solve. Usually, we will need to simplify one or both sides of an equation before using the Subtraction or Addition Properties of Equality. You should always simplify as much as possible before trying to isolate the variable.

Example:
Exercise:

Problem: Solve: $3x - 7 - 2x - 4 = 1$.

Solution:
Solution

The left side of the equation has an expression that we should simplify before trying to isolate the variable.

	$3x-7-2x-4=1$
Rearrange the terms, using the Commutative Property of Addition.	$3x-2x-7-4=1$
Combine like terms.	$x-11=1$
Add 11 to both sides to isolate x.	$x-11+11=1+11$
Simplify.	$x=12$
Check. Substitute $x=12$ into the original equation. $3x-7-2x-4=1$ $3(12)-7-2(12)-4=1$ $36-7-24-4=1$ $29-24-4=1$ $5-4=1$ $1=1$ ✓	

The solution checks.

Note:
Exercise:

Problem: Solve: $8y - 4 - 7y - 7 = 4$.

Solution:

y = 15

Note:
Exercise:

Problem: Solve: $6z + 5 - 5z - 4 = 3$.

Solution:

z = 2

Example:
Exercise:

Problem: Solve: $3(n - 4) - 2n = -3$.

Solution:
Solution

The left side of the equation has an expression that we should simplify.

	$3(n-4)-2n=-3$
Distribute on the left.	$3n-12-2n=-3$
Use the Commutative Property to rearrange terms.	$3n-2n-12=-3$
Combine like terms.	$n-12=-3$
Isolate n using the Addition Property of Equality.	$n-12+12=-3+12$
Simplify.	$n=9$
Check. Substitute $n=9$ into the original equation. $3(n-4)-2n=-3$ $3(9-4)-2\cdot 9=-3$ $3(5)-18=-3$ $15-18=-3$ $-3=-3$ ✓	

The solution checks.

Note:
Exercise:

Problem: Solve: $5(p-3)-4p=-10$.

Solution:

$p=5$

Note:
Exercise:

Problem: Solve: $4(q+2)-3q=-8$.

Solution:

$q=-16$

Example:
Exercise:

Problem: Solve: $2(3k-1)-5k=-2-7$.

Solution:
Solution

Both sides of the equation have expressions that we should simplify before we isolate the variable.

	$2(3k-1)-5k=-2-7$
Distribute on the left, subtract on the right.	$6k-2-5k=-9$
Use the Commutative Property of Addition.	$6k-5k-2=-9$
Combine like terms.	$k-2=-9$
Undo subtraction by using the Addition Property of Equality.	$k-2+2=-9+2$
Simplify.	$k=-7$
Check. Let $k=-7$. $2(3k-1)-5k=-2-7$ $2(3(-7)-1)-5(-7)=-2-7$ $2(-21-1)-5(-7)=-9$ $2(-22)+35=-9$ $-44+35=-9$ $-9=-9$ ✓	

The solution checks.	

Note:
Exercise:

Problem: Solve: $4(2h-3)-7h=-6-7$.

Solution:

$h=-1$

Note:
Exercise:

Problem: Solve: $2(5x+2)-9x=-2+7$.

Solution:

$x=1$

Translate an Equation and Solve

In previous chapters, we translated word sentences into equations. The first step is to look for the word (or words) that translate(s) to the equal sign. [link] reminds us of some of the words that translate to the equal sign.

Equals (=)						
is	is equal to	is the same as	the result is	gives	was	will be

Let’s review the steps we used to translate a sentence into an equation.

Note:
Translate a word sentence to an algebraic equation.

Locate the "equals" word(s). Translate to an equal sign.
Translate the words to the left of the "equals" word(s) into an algebraic expression.
Translate the words to the right of the "equals" word(s) into an algebraic expression.

Now we are ready to try an example.

Example:
Exercise:

Problem: Translate and solve: five more than x is equal to 26.

Solution:
Solution

Translate.	$\underbrace{\text{Five more than } x}_{x+5} \quad \underbrace{\text{is equal to}}_{=} \quad \underbrace{26}_{26}$
Subtract 5 from both sides.	$x+5-5=26-5$
Simplify.	$x=21$
Check: Is 26 five more than 21? $21+5\overset{?}{=}26$ $26=26\checkmark$ The solution checks.	

Note:
Exercise:

Problem: Translate and solve: Eleven more than x is equal to 41.

Solution:

$x + 11 = 41$; $x = 30$

Note:
Exercise:

Problem: Translate and solve: Twelve less than y is equal to 51.

Solution:

$y - 12 = 51; y = 63$

Example:
Exercise:

Problem: Translate and solve: The difference of $5p$ and $4p$ is 23.

Solution:
Solution

Translate.	$\underbrace{\text{The difference of } 5p \text{ and } 4p}_{5p - 4p}\quad\underbrace{\text{is}}_{=}\quad\underbrace{23}_{23}$
Simplify.	$p = 23$
Check: $5p - 4p = 23$	

$5(23) - 4(23) \overset{?}{=} 23$ $115 - 22 \overset{?}{=} 23$ $23 = 23 \checkmark$	
The solution checks.	

Note:
Exercise:

Problem: Translate and solve: The difference of $4x$ and $3x$ is 14.

Solution:

$4x - 3x = 14$; $x = 14$

Note:
Exercise:

Problem: Translate and solve: The difference of $7a$ and $6a$ is -8.

Solution:

$7a - 6a = -8; a = -8$

Translate and Solve Applications

In most of the application problems we solved earlier, we were able to find the quantity we were looking for by simplifying an algebraic expression. Now we will be using equations to solve application problems. We’ll start by restating the problem in just one sentence, assign a variable, and then translate the sentence into an equation to solve. When assigning a variable, choose a letter that reminds you of what you are looking for.

Example:
Exercise:

Problem:

The Robles family has two dogs, Buster and Chandler. Together, they weigh 71 pounds.

Chandler weighs 28 pounds. How much does Buster weigh?

Solution:
Solution

Read the problem carefully.	
Identify what you are asked to find, and choose a variable to represent it.	How much does Buster weigh?

	Let b = Buster's weight
Write a sentence that gives the information to find it.	Buster's weight plus Chandler's weight equals 71 pounds.
We will restate the problem, and then include the given information.	Buster's weight plus 28 equals 71.
Translate the sentence into an equation, using the variable b.	$b+28=71$
Solve the equation using good algebraic techniques.	$b+28-28=71-28$ $b=43$
Check the answer in the problem and make sure it makes sense.	
Is 43 pounds a reasonable weight for a dog? Yes. Does Buster's weight plus Chandler's weight equal 71 pounds?	
$43+28\overset{?}{=}71$	
$71=71$ ✓	
Write a complete sentence that answers the question, "How much does Buster weigh?"	Buster weighs 43 pounds

Note:
Exercise:

Problem:

Translate into an algebraic equation and solve: The Pappas family has two cats, Zeus and Athena. Together, they weigh 13 pounds. Zeus weighs 6 pounds. How much does Athena weigh?

Solution:

$a + 6 = 13$; Athena weighs 7 pounds.

Note:
Exercise:

Problem:

Translate into an algebraic equation and solve: Sam and Henry are roommates. Together, they have 68 books. Sam has 26 books. How many books does Henry have?

Solution:

$26 + h = 68$; Henry has 42 books.

Note:
Devise a problem-solving strategy.

Read the problem. Make sure you understand all the words and ideas. Identify what you are looking for.

Name what you are looking for. Choose a variable to represent that quantity.
Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
Solve the equation using good algebra techniques.
Check the answer in the problem and make sure it makes sense.
Answer the question with a complete sentence.

Example:
Exercise:

Problem:

Shayla paid $\$24,575$ for her new car. This was $\$875$ less than the sticker price. What was the sticker price of the car?

Solution:
Solution

What are you asked to find?	"What was the sticker price of the car?"
Assign a variable.	Let $s =$ the sticker price of the car.
Write a sentence that gives the information to find it.	\$24,575 is \$875 less than the sticker price \$24,575 is \$875 less than s

Translate into an equation.	$24{,}575 = s - 875$
Solve.	$24{,}575 + 875 = s - 875 + 875$ $24{,}575 = s$
Check:	
Is $875 less than $25,450 equal to $24,575?	
$25{,}450 - 875 \overset{?}{=} 24{,}575$	
$24{,}575 = 24{,}575$ ✓	
Write a sentence that answers the question.	The sticker price was $25,450.

Note:
Exercise:

Problem:

Translate into an algebraic equation and solve: Eddie paid $19,875 for his new car. This was $1,025 less than the sticker price. What was the sticker price of the car?

Solution:

19,875 = s − 1025; the sticker price is $20,900.

Note:
Exercise:

Problem:

Translate into an algebraic equation and solve: The admission price for the movies during the day is $\$7.75$. This is $\$3.25$ less than the price at night. How much does the movie cost at night?

Solution:

7.75 = n − 3.25; the price at night is $11.00.

Note: The *Links to Literacy* activity, "The 100-pound Problem", will provide you with another view of the topics covered in this section.

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Solving One Step Equations By Addition and Subtraction
- Solve One Step Equations By Add and Subtract Whole Numbers (Variable on Left)
- Solve One Step Equations By Add and Subtract Whole Numbers (Variable on Right)

Key Concepts

- **Determine whether a number is a solution to an equation.**

  Substitute the number for the variable in the equation.
  Simplify the expressions on both sides of the equation.
  Determine whether the resulting equation is true.

  If it is true, the number is a solution.
  If it is not true, the number is not a solution.
- **Subtraction and Addition Properties of Equality**
  - **Subtraction Property of Equality**
    For all real numbers a, b, and c,
    if $a = b$ then $a - c = b - c$.
  - **Addition Property of Equality**
    For all real numbers a, b, and c,
    if $a = b$ then $a + c = b + c$.

- **Translate a word sentence to an algebraic equation.**

  Locate the “equals” word(s). Translate to an equal sign.
  Translate the words to the left of the “equals” word(s) into an algebraic expression.
  Translate the words to the right of the “equals” word(s) into an algebraic expression.

- **Problem-solving strategy**

  Read the problem. Make sure you understand all the words and ideas.
  Identify what you are looking for.
  Name what you are looking for. Choose a variable to represent that quantity.
  Translate into an equation. It may be helpful to restate the problem in one sentence with all the important information. Then, translate the English sentence into an algebra equation.
  Solve the equation using good algebra techniques.
  Check the answer in the problem and make sure it makes sense.
  Answer the question with a complete sentence.

Practice Makes Perfect

Solve Equations Using the Subtraction and Addition Properties of Equality

In the following exercises, determine whether the given value is a solution to the equation.

Exercise:

Problem: Is $y = \frac{1}{3}$ a solution of $4y + 2 = 10y$?

Solution:

yes

Exercise:

Problem: Is $x = \frac{3}{4}$ a solution of $5x + 3 = 9x$?

Exercise:

Problem: Is $u = -\frac{1}{2}$ a solution of $8u - 1 = 6u$?

Solution:

no

Exercise:

Problem: Is $v = -\frac{1}{3}$ a solution of $9v - 2 = 3v$?

In the following exercises, solve each equation.

Exercise:

Problem: $x + 7 = 12$

Solution:

$x = 5$

Exercise:

Problem: $y + 5 = -6$

Exercise:

Problem: $b + \frac{1}{4} = \frac{3}{4}$

Solution:

$b = \frac{1}{2}$

Exercise:

Problem: $a + \frac{2}{5} = \frac{4}{5}$

Exercise:

Problem: $p + 2.4 = -9.3$

Solution:

$p = -11.7$

Exercise:

Problem: $m + 7.9 = 11.6$

Exercise:

Problem: $a - 3 = 7$

Solution:

$a = 10$

Exercise:

Problem: $m-8=-20$

Exercise:

Problem: $x-\frac{1}{3}=2$

Solution:

$x=\frac{7}{3}$

Exercise:

Problem: $x-\frac{1}{5}=4$

Exercise:

Problem: $y-3.8=10$

Solution:

$y = 13.8$

Exercise:

Problem: $y-7.2=5$

Exercise:

Problem: $x-15=-42$

Solution:

$x = -27$

Exercise:

Problem: $z+5.2=-8.5$

Exercise:

Problem: $q+\frac{3}{4}=\frac{1}{2}$

Solution:

$q=-\frac{1}{4}$

Exercise:

Problem: $p-\frac{2}{5}=\frac{2}{3}$

Exercise:

Problem: $y-\frac{3}{4}=\frac{3}{5}$

Solution:

$y=\frac{27}{20}$

Solve Equations that Need to be Simplified

In the following exercises, solve each equation.

Exercise:

Problem: $c+3-10=18$

Exercise:

Problem: $m+6-8=15$

Solution:

17

Exercise:

Problem: $9x+5-8x+14=20$

Exercise:

Problem: $6x+8-5x+16=32$

Solution:

8

Exercise:

Problem: $-6x-11+7x-5=-16$

Exercise:

Problem: $-8n-17+9n-4=-41$

Solution:

−20

Exercise:

Problem: $3(y-5)-2y=-7$

Exercise:

Problem: $4(y-2)-3y=-6$

Solution:

2

Exercise:

Problem: $8(u+1.5)-7u=4.9$

Exercise:

Problem: $5(w+2.2)-4w=9.3$

Solution:

-1.7

Exercise:

Problem: $-5(y-2)+6y=-7+4$

Exercise:

Problem: $-8(x-1)+9x=-3+9$

Solution:

-2

Exercise:

Problem: $3(5n-1)-14n+9=1-2$

Exercise:

Problem: $2(8m+3)-15m-4=3-5$

Solution:

-4

Exercise:

Problem: $-(j+2)+2j-1=5$

Exercise:

Problem: $-(k+7)+2k+8=7$

Solution:

6

Exercise:

Problem: $6a-5(a-2)+9=-11$

Exercise:

Problem: $8c-7(c-3)+4=-16$

Solution:

−41

Exercise:

Problem: $8(4x+5)-5(6x)-x=53$

Exercise:

Problem: $6(9y-1)-10(5y)-3y=22$

Solution:

28

Translate to an Equation and Solve

In the following exercises, translate to an equation and then solve.

Exercise:

Problem: Five more than x is equal to 21.

Exercise:

Problem: The sum of x and -5 is 33.

Solution:

$x + (-5) = 33; x = 38$

Exercise:

Problem: Ten less than m is -14.

Exercise:

Problem: Three less than y is -19.

Solution:

$y - 3 = -19; y = -16$

Exercise:

Problem: The sum of y and -3 is 40.

Exercise:

Problem: Eight more than p is equal to 52.

Solution:

$p + 8 = 52; p = 44$

Exercise:

Problem: The difference of $9x$ and $8x$ is 17.

Exercise:

Problem: The difference of $5c$ and $4c$ is 60.

Solution:

$5c - 4c = 60; 60$

Exercise:

Problem: The difference of n and $\frac{1}{6}$ is $\frac{1}{2}$.

Exercise:

Problem: The difference of f and $\frac{1}{3}$ is $\frac{1}{12}$.

Solution:

$f - \frac{1}{3} = \frac{1}{12}; \frac{5}{12}$

Exercise:

Problem: The sum of $-4n$ and $5n$ is -32.

Exercise:

Problem: The sum of $-9m$ and $10m$ is -25.

Solution:

$-9m + 10m = -25; m = -25$

Translate and Solve Applications

In the following exercises, translate into an equation and solve.

Exercise:

Problem:

Pilar drove from home to school and then to her aunt’s house, a total of 18 miles. The distance from Pilar’s house to school is 7 miles. What is the distance from school to her aunt’s house?

Exercise:

Problem:

Jeff read a total of 54 pages in his English and Psychology textbooks. He read 41 pages in his English textbook. How many pages did he read in his Psychology textbook?

Solution:

Let p equal the number of pages read in the Psychology book $41 + p = 54$. Jeff read pages in his Psychology book.

Exercise:

Problem:

Pablo’s father is 3 years older than his mother. Pablo’s mother is 42 years old. How old is his father?

Exercise:

Problem:

Eva’s daughter is 5 years younger than her son. Eva’s son is 12 years old. How old is her daughter?

Solution:

Let d equal the daughter’s age. $d = 12 - 5$. Eva’s daughter’s age is 7 years old.

Exercise:

Problem:

Allie weighs 8 pounds less than her twin sister Lorrie. Allie weighs 124 pounds. How much does Lorrie weigh?

Exercise:

Problem:

For a family birthday dinner, Celeste bought a turkey that weighed 5 pounds less than the one she bought for Thanksgiving. The birthday dinner turkey weighed 16 pounds. How much did the Thanksgiving turkey weigh?

Solution:

21 pounds

Exercise:

Problem:

The nurse reported that Tricia's daughter had gained 4.2 pounds since her last checkup and now weighs 31.6 pounds. How much did Tricia's daughter weigh at her last checkup?

Exercise:

Problem:

Connor's temperature was 0.7 degrees higher this morning than it had been last night. His temperature this morning was 101.2 degrees. What was his temperature last night?

Solution:

100.5 degrees

Exercise:

Problem:

Melissa's math book cost $\$22.85$ less than her art book cost. Her math book cost $\$93.75$. How much did her art book cost?

Exercise:

Problem:

Ron’s paycheck this week was $\$17.43$ less than his paycheck last week. His paycheck this week was $\$103.76$. How much was Ron’s paycheck last week?

Solution:

$121.19

Everyday Math

Exercise:

Problem:

Baking Kelsey needs $\frac{2}{3}$ cup of sugar for the cookie recipe she wants to make. She only has $\frac{1}{4}$ cup of sugar and will borrow the rest from her neighbor. Let s equal the amount of sugar she will borrow. Solve the equation $\frac{1}{4}+s=\frac{2}{3}$ to find the amount of sugar she should ask to borrow.

Exercise:

Problem:

Construction Miguel wants to drill a hole for a $\frac{5}{8}$ -inch screw. The screw should be $\frac{1}{12}$ inch larger than the hole. Let d equal the size of the hole he should drill. Solve the equation $d+\frac{1}{12}=\frac{5}{8}$ to see what size the hole should be.

Solution:

$d=\frac{13}{24}$

Writing Exercises

Exercise:

Problem:

Is -18 a solution to the equation $3x = 16 - 5x$? How do you know?

Exercise:

Problem:

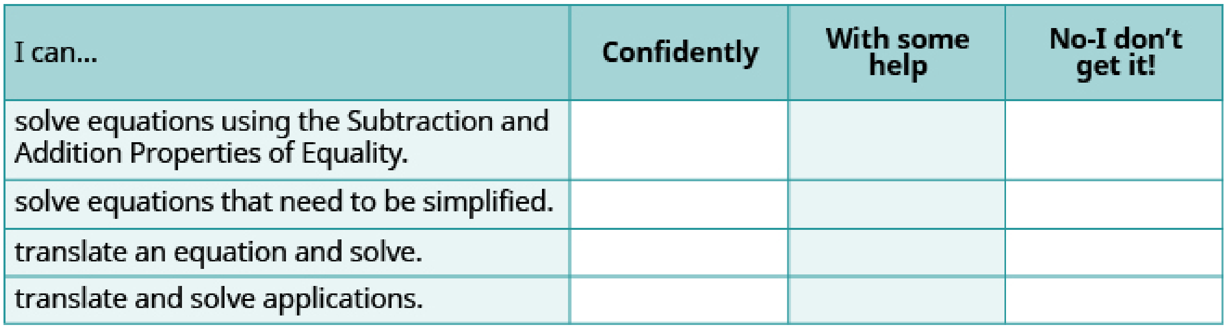
Write a word sentence that translates the equation $y - 18 = 41$ and then make up an application that uses this equation in its solution.

Solution:

Answers will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
solve equations using the Subtraction and Addition Properties of Equality.			
solve equations that need to be simplified.			
translate an equation and solve.			
translate and solve applications.			

ⓑ If most of your checks were:

…confidently. Congratulations! You have achieved the objectives in this section. Reflect on the study skills you used so that you can continue to use them. What did you do to become confident of your ability to do these things? Be specific.

…with some help. This must be addressed quickly because topics you do not master become potholes in your road to success. In math, every topic builds upon previous work. It is important to make sure you have a strong foundation before you move on. Who can you ask for help? Your fellow classmates and instructor are good resources. Is there a place on campus where math tutors are available? Can your study skills be improved?

…no—I don’t get it! This is a warning sign and you must not ignore it. You should get help right away or you will quickly be overwhelmed. See your instructor as soon as you can to discuss your situation. Together you can come up with a plan to get you the help you need.

Glossary

solution of an equation

A solution of an equation is a value of a variable that makes a true statement when substituted into the equation.

Solve Equations Using the Division and Multiplication Properties of Equality
By the end of this section, you will be able to:

- Solve equations using the Division and Multiplication Properties of Equality
- Solve equations that need to be simplified

Note:
Before you get started, take this readiness quiz.

1. Simplify: $-7\left(\frac{1}{-7}\right)$.
   If you missed this problem, review [link].
2. What is the reciprocal of $-\frac{3}{8}$?
   If you missed this problem, review [link].
3. Evaluate $9x+2$ when $x=-3$.
   If you missed this problem, review [link].

Solve Equations Using the Division and Multiplication Properties of Equality

We introduced the Multiplication and Division Properties of Equality in Solve Equations Using Integers; The Division Property of Equality and Solve Equations with Fractions. We modeled how these properties worked using envelopes and counters and then applied them to solving equations (See Solve Equations Using Integers; The Division Property of Equality). We restate them again here as we prepare to use these properties again.

Note:
Division and Multiplication Properties of Equality

Division Property of Equality: For all real numbers a, b, c, and $c \neq 0$, if $a = b$, then $\frac{a}{c} = \frac{b}{c}$.
Multiplication Property of Equality: For all real numbers a, b, c, if $a = b$, then $ac = bc$.

When you divide or multiply both sides of an equation by the same quantity, you still have equality.

Let’s review how these properties of equality can be applied in order to solve equations. Remember, the goal is to ‘undo’ the operation on the variable. In the example below the variable is multiplied by 4, so we will divide both sides by 4 to ‘undo’ the multiplication.

Example:
Exercise:

Problem: Solve: $4x = -28$.

Solution:
Solution

We use the Division Property of Equality to divide both sides by 4.

	$4x = -28$
Divide both sides by 4 to undo the multiplication.	$\frac{4x}{4} = \frac{-28}{4}$

Simplify.	$x = -7$
Check your answer. Let $x = -7$.	
$4x = -28$	
$4(-7) \overset{?}{=} -28$	
$-28 = -28$ ✓	

Since this is a true statement, $x = -7$ is a solution to $4x = -28$.

Note:
Exercise:

Problem: Solve: $3y = -48$.

Solution:

$y = -16$

Note:
Exercise:

Problem: Solve: $4z = -52$.

Solution:

$z = -13$

In the previous example, to ‘undo’ multiplication, we divided. How do you think we ‘undo’ division?

Example:
Exercise:

Problem: Solve: $\frac{a}{-7} = -42$.

Solution:
Solution

Here a is divided by -7. We can multiply both sides by -7 to isolate a.

	$\frac{a}{-7} = -42$
Multiply both sides by -7.	$-7\left(\frac{a}{-7}\right) = -7(-42)$

	$\frac{-7a}{-7} = 294$
Simplify.	$a = 294$
Check your answer. Let $a = 294$.	
$\frac{a}{-7} = -42$	
$\frac{294}{-7} \overset{?}{=} -42$	
$-42 = -42$ ✓	

Note:
Exercise:

Problem: Solve: $\frac{b}{-6} = -24$.

Solution:

$b = 144$

Note:
Exercise:

Problem: Solve: $\frac{c}{-8} = -16$.

Solution:

$c = 128$

Example:
Exercise:

Problem: Solve: $-r = 2$.

Solution:
Solution

Remember $-r$ is equivalent to $-1r$.

	$-r = 2$
Rewrite $-r$ as $-1r$.	$-1r = 2$
Divide both sides by -1.	$\frac{-1r}{-1} = \frac{2}{-1}$

		$r = -2$
Check.	$-r = 2$	
Substitute $r = -2$	$-(-2) \overset{?}{=} 2$	
Simplify.	$2 = 2 ✓$	

In Solve Equations with Fractions, we saw that there are two other ways to solve $-r = 2$.

We could multiply both sides by -1.

We could take the opposite of both sides.

Note:
Exercise:

Problem: Solve: $-k = 8$.

Solution:

$k = -8$

Note:

Exercise:

Problem: Solve: $-g = 3$.

Solution:

$g = -3$

Example:
Exercise:

Problem: Solve: $\frac{2}{3}x = 18$.

Solution:
Solution

Since the product of a number and its reciprocal is 1, our strategy will be to isolate x by multiplying by the reciprocal of $\frac{2}{3}$.

	$\frac{2}{3}x = 18$
Multiply by the reciprocal of $\frac{2}{3}$.	$\frac{3}{2} \cdot \frac{2}{3}x = \frac{3}{2} \cdot 18$
Reciprocals multiply to one.	

	$1x = \frac{3}{2} \cdot \frac{18}{1}$
Multiply.	$x = 27$
Check your answer. Let $x = 27$	
$\frac{2}{3}x = 18$	
$\frac{2}{3} \cdot 27 \overset{?}{=} 18$	
$18 = 18$ ✓	

Notice that we could have divided both sides of the equation $\frac{2}{3}x = 18$ by $\frac{2}{3}$ to isolate x. While this would work, multiplying by the reciprocal requires fewer steps.

Note:
Exercise:

Problem: Solve: $\frac{2}{5}n = 14$.

Solution:

$n = 35$

Note:
Exercise:

Problem: Solve: $\frac{5}{6}y = 15$.

Solution:

$y = 18$

Solve Equations That Need to be Simplified

Many equations start out more complicated than the ones we’ve just solved. First, we need to simplify both sides of the equation as much as possible

Example:
Exercise:

Problem: Solve: $8x + 9x - 5x = -3 + 15$.

Solution:
Solution

Start by combining like terms to simplify each side.

	$8x + 9x - 5x = -3 + 15$
Combine like terms.	$12x = 12$
Divide both sides by 12 to isolate x.	$\frac{12x}{12} = \frac{12}{12}$
Simplify.	$x = 1$
Check your answer. Let $x = 1$	
$8x + 9x - 5x = -3 + 15$	
$8 \cdot 1 + 9 \cdot 1 - 5 \cdot 1 \overset{?}{=} -3 + 15$	
$8 + 9 - 5 \overset{?}{=} -3 + 15$	
$12 = 12 \checkmark$	

Note:
Exercise:

Problem: Solve: $7x + 6x - 4x = -8 + 26$.

Solution:

$x = 2$

Note:
Exercise:

Problem: Solve: $11n - 3n - 6n = 7 - 17$.

Solution:

$n = -5$

Example:
Exercise:

Problem: Solve: $11 - 20 = 17y - 8y - 6y$.

Solution:
Solution

Simplify each side by combining like terms.

	$11 - 20 = 17y - 8y - 6y$

Simplify each side.	$-9 = 3y$
Divide both sides by 3 to isolate y.	$\frac{-9}{3} = \frac{3y}{3}$
Simplify.	$-3 = y$
Check your answer. Let $y = -3$	
$11 - 20 = 17y - 8y - 6y$	
$11 - 20 \overset{?}{=} 17(-3) - 8(-3) - 6(-3)$	
$11 - 20 \overset{?}{=} -51 + 24 + 18$	
$-9 = -9$ ✓	

Notice that the variable ended up on the right side of the equal sign when we solved the equation. You may prefer to take one more step to write the solution with the variable on the left side of the equal sign.

Note:
Exercise:

Problem: Solve: $18-27=15c-9c-3c$.

Solution:

$c=-3$

Note:
Exercise:

Problem: Solve: $18-22=12x-x-4x$.

Solution:

$x=-\frac{4}{7}$

Example:
Exercise:

Problem: Solve: $-3(n-2)-6=21$.

Solution:
Solution

Remember—always simplify each side first.

	$-3(n-2)-6=21$
Distribute.	$-3n+6-6=21$
Simplify.	$-3n=21$
Divide both sides by -3 to isolate n.	$\frac{-3n}{-3}=\frac{21}{-3}$ $n=-7$
Check your answer. Let $n=-7$.	
$-3(n-2)-6=21$	
$-3(-7-2)-6\overset{?}{=}21$	
$-3(-9)-6\overset{?}{=}21$	
$27-6\overset{?}{=}21$	

$21 = 21 \checkmark$

Note:
Exercise:

Problem: Solve: $-4(n-2)-8=24$.

Solution:

$n = -6$

Note:
Exercise:

Problem: Solve: $-6(n-2)-12=30$.

Solution:

$n = -5$

Note: The *Links to Literacy* activity, "Everybody Wins" will provide you with another view of the topics covered in this section.

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Solving One Step Equation by Mult/Div. Integers (Var on Left)
- Solving One Step Equation by Mult/Div. Integers (Var on Right)
- Solving One Step Equation in the Form: $-x = -a$

Key Concepts

- **Division and Multiplication Properties of Equality**
  - **Division Property of Equality:** For all real numbers a, b, c, and $c \neq 0$, if $a = b$, then $ac = bc$.
  - **Multiplication Property of Equality:** For all real numbers a, b, c, if $a = b$, then $ac = bc$.

Practice Makes Perfect

Solve Equations Using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation for the variable using the Division Property of Equality and check the solution.

Exercise:

Problem: $8x = 32$

Exercise:

Problem: $7p = 63$

Solution:

9

Exercise:

Problem: $-5c = 55$

Exercise:

Problem: $-9x = -27$

Solution:

3

Exercise:

Problem: $-90 = 6y$

Exercise:

Problem: $-72 = 12y$

Solution:

-6

Exercise:

Problem: $-16p = -64$

Exercise:

Problem: $-8m = -56$

Solution:

7

Exercise:

Problem: $0.25z = 3.25$

Exercise:

Problem: $0.75a = 11.25$

Solution:

15

Exercise:

Problem: $-3x = 0$

Exercise:

Problem: $4x = 0$

Solution:

0

In the following exercises, solve each equation for the variable using the Multiplication Property of Equality and check the solution.

Exercise:

Problem: $\frac{x}{4} = 15$

Exercise:

Problem: $\frac{z}{2} = 14$

Solution:

28

Exercise:

Problem: $-20 = \frac{q}{-5}$

Exercise:

Problem: $\frac{c}{-3} = -12$

Solution:

36

Exercise:

Problem: $\frac{y}{9} = -6$

Exercise:

Problem: $\frac{q}{6} = -8$

Solution:

−48

Exercise:

Problem: $\frac{m}{-12} = 5$

Exercise:

Problem: $-4 = \frac{p}{-20}$

Solution:

80

Exercise:

Problem: $\frac{2}{3}y = 18$

Exercise:

Problem: $\frac{3}{5}r = 15$

Solution:

25

Exercise:

Problem: $-\frac{5}{8}w = 40$

Exercise:

Problem: $24 = -\frac{3}{4}x$

Solution:

−32

Exercise:

Problem: $-\frac{2}{5} = \frac{1}{10}a$

Exercise:

Problem: $-\frac{1}{3}q = -\frac{5}{6}$

Solution:

5/2

Solve Equations That Need to be Simplified

In the following exercises, solve the equation.
Exercise:

Problem: $8a + 3a - 6a = -17 + 27$

Exercise:

Problem: $6y - 3y + 12y = -43 + 28$

Solution:

$y = -1$

Exercise:

Problem: $-9x - 9x + 2x = 50 - 2$

Exercise:

Problem: $-5m + 7m - 8m = -6 + 36$

Solution:

$m = -5$

Exercise:

Problem: $100 - 16 = 4p - 10p - p$

Exercise:

Problem: $-18 - 7 = 5t - 9t - 6t$

Solution:

$t = \frac{5}{2}$

Exercise:

Problem: $\frac{7}{8}n - \frac{3}{4}n = 9 + 2$

Exercise:

Problem: $\frac{5}{12}q + \frac{1}{2}q = 25 - 3$

Solution:

$q = 24$

Exercise:

Problem: $0.25d + 0.10d = 6 - 0.75$

Exercise:

Problem: $0.05p - 0.01p = 2 + 0.24$

Solution:

$p = 56$

Everyday Math

Exercise:

Problem:

Balloons Ramona bought 18 balloons for a party. She wants to make 3 equal bunches. Find the number of balloons in each bunch, b, by solving the equation $3b = 18$.

Exercise:

Problem:

Teaching Connie’s kindergarten class has 24 children. She wants them to get into 4 equal groups. Find the number of children in each group, g, by solving the equation $4g = 24$.

Solution:

6 children

Exercise:

Problem:

Ticket price Daria paid $\$36.25$ for 5 children's tickets at the ice skating rink. Find the price of each ticket, p, by solving the equation $5p = 36.25$.

Exercise:

Problem:

Unit price Nishant paid $\$12.96$ for a pack of 12 juice bottles. Find the price of each bottle, b, by solving the equation $12b = 12.96$.

Solution:

$1.08

Exercise:

Problem:

Fuel economy Tania's SUV gets half as many miles per gallon (mpg) as her husband's hybrid car. The SUV gets 18 mpg. Find the miles per gallons, m, of the hybrid car, by solving the equation $\frac{1}{2}m = 18$.

Exercise:

Problem:

Fabric The drill team used 14 yards of fabric to make flags for one-third of the members. Find how much fabric, f, they would need to make flags for the whole team by solving the equation $\frac{1}{3}f = 14$.

Solution:

42 yards

Writing Exercises

Exercise:

Problem:

Frida started to solve the equation $-3x = 36$ by adding 3 to both sides. Explain why Frida's method will result in the correct solution.

Exercise:

Problem:

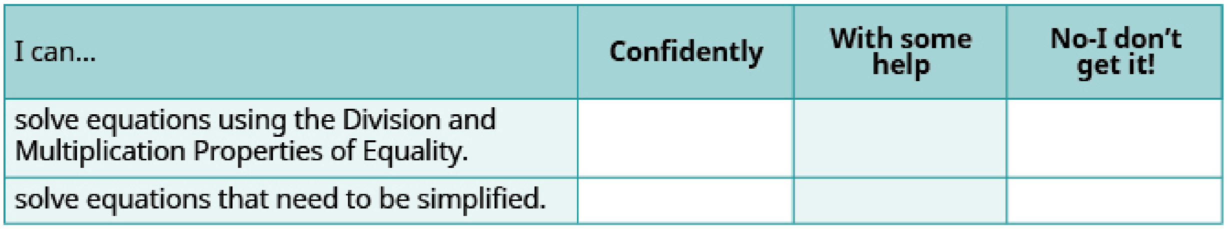
Emiliano thinks $x = 40$ is the solution to the equation $\frac{1}{2}x = 80$. Explain why he is wrong.

Solution:

Answer will vary.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
solve equations using the Division and Multiplication Properties of Equality.			
solve equations that need to be simplified.			

ⓑ After reviewing this checklist, what will you do to become confident for all objectives?

Solve Equations with Variables and Constants on Both Sides

By the end of this section, you will be able to:

- Solve an equation with constants on both sides
- Solve an equation with variables on both sides
- Solve an equation with variables and constants on both sides
- Solve equations using a general strategy

Note:

Before you get started, take this readiness quiz.

1. Simplify: $4y-9+9$.
   If you missed this problem, review [link].
2. Solve: $y+12=16$.
   If you missed this problem, review [link].
3. Solve: $-3y=63$.
   If you missed this problem, review [link].

Solve an Equation with Constants on Both Sides

You may have noticed that in all the equations we have solved so far, all the variable terms were on only one side of the equation with the constants on the other side. This does not happen all the time—so now we'll see how to solve equations where the variable terms and/or constant terms are on both sides of the equation.

Our strategy will involve choosing one side of the equation to be the variable side, and the other side of the equation to be the constant side. Then, we will use the Subtraction and Addition Properties of Equality, step by step, to get all the variable terms together on one side of the equation and the constant terms together on the other side.

By doing this, we will transform the equation that started with variables and constants on both sides into the form $ax = b$. We already know how to solve equations of this form by using the Division or Multiplication Properties of Equality.

Example:
Exercise:

Problem: Solve: $4x + 6 = -14$.

Solution:
Solution

In this equation, the variable is only on the left side. It makes sense to call the left side the variable side. Therefore, the right side will be the constant side. We'll write the labels above the equation to help us remember what goes where.

	$\overset{\text{variable}}{4x + 6} = \overset{\text{constant}}{-14}$
Since the left side is the variable side, the 6 is out of place. We must "undo" adding 6 by subtracting 6, and to keep the equality we must subtract 6 from both sides. Use the Subtraction Property of Equality.	$4x + 6 - 6 = -14 - 6$
Simplify.	$4x = -20$
Now all the xs are on the left and the constant on the right.	

Use the Division Property of Equality.		$\frac{4x}{4}=\frac{-20}{4}$
Simplify.		$x=-5$
Check:	$4x+6=-14$	
Let $x=-5$.	$4(-5)+6=-14$	
	$-20+6=-14$	
	$-14=-14$ ✓	

Note:
Exercise:

Problem: Solve: $3x+4=-8$.

Solution:

$x=-4$

Note:

Exercise:

Problem: Solve: $5a + 3 = -37$.

Solution:

$a = -8$

Example:
Exercise:

Problem: Solve: $2y - 7 = 15$.

Solution:
Solution

Notice that the variable is only on the left side of the equation, so this will be the variable side and the right side will be the constant side. Since the left side is the variable side, the 7 is out of place. It is subtracted from the $2y$, so to 'undo' subtraction, add 7 to both sides.

	$\overset{\text{variable}}{2y - 7} = \overset{\text{constant}}{15}$
Add 7 to both sides.	$2y - 7 + 7 = 15 + 7$
Simplify.	$2y = 22$

The variables are now on one side and the constants on the other.		
Divide both sides by 2.		$\frac{2y}{2} = \frac{22}{2}$
Simplify.		$y = 11$
Check:	$2y - 7 = 15$	
Substitute: $y = 11$.	$2 \cdot 11 - 7 \overset{?}{=} 15$	
	$22 - 7 \overset{?}{=} 15$	
	$15 = 15$ ✓	

Note:
Exercise:

Problem: Solve: $5y - 9 = 16$.

Solution:

$y = 5$

Note:
Exercise:

Problem: Solve: $3m - 8 = 19$.

Solution:

$m = 9$

Solve an Equation with Variables on Both Sides

What if there are variables on both sides of the equation? We will start like we did above—choosing a variable side and a constant side, and then use the Subtraction and Addition Properties of Equality to collect all variables on one side and all constants on the other side. Remember, what you do to the left side of the equation, you must do to the right side too.

Example:
Exercise:

Problem: Solve: $5x = 4x + 7$.

Solution:
Solution

Here the variable, x, is on both sides, but the constants appear only on the right side, so let’s make the right side the “constant” side. Then the left side will be the “variable” side.

		$\overset{\text{variable}}{5x} = 4x + \overset{\text{constant}}{7}$
We don't want any variables on the right, so subtract the $4x$.		$5x - 4x = 4x - 4x + 7$
Simplify.		$x = 7$
We have all the variables on one side and the constants on the other. We have solved the equation.		
Check:	$5x = 4x + 7$	
Substitute 7 for x.	$5(7) \stackrel{?}{=} 4(7) + 7$	
	$35 \stackrel{?}{=} 28 + 7$	
	$35 = 35$ ✓	

Note:
Exercise:

Problem: Solve: $6n = 5n + 10$.

Solution:

$n = 10$

Note:
Exercise:

Problem: Solve: $-6c = -7c + 1$.

Solution:

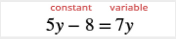
$c = 1$

Example:
Exercise:

Problem: Solve: $5y - 8 = 7y$.

Solution:
Solution

The only constant, -8, is on the left side of the equation and variable, y, is on both sides. Let’s leave the constant on the left and collect the variables to the right.

Subtract $5y$ from both sides.	$5y - 5y - 8 = 7y - 5y$
Simplify.	$-8 = 2y$
We have the variables on the right and the constants on the left. Divide both sides by 2.	$\frac{-8}{2} = \frac{2y}{2}$
Simplify.	$-4 = y$
Rewrite with the variable on the left.	$y = -4$
Check: Let $y = -4$.	
$5y - 8 = 7y$	
$5(-4) - 8 \overset{?}{=} 7(-4)$	
$-20 - 8 \overset{?}{=} -28$	
$-28 = -28$ ✓	

Note:
Exercise:

Problem: Solve: $3p - 14 = 5p$.

Solution:

$p = -7$

Note:
Exercise:

Problem: Solve: $8m + 9 = 5m$.

Solution:

$m = -3$

Example:
Exercise:

Problem: Solve: $7x = -x + 24$.

Solution:
Solution

The only constant, 24, is on the right, so let the left side be the variable side.

	$\overset{\text{variable side}}{7x} = \overset{\text{constant side}}{-x + 24}$
Remove the $-x$ from the right side by adding x to both sides.	$7x + x = -x + x + 24$
Simplify.	$8x = 24$
All the variables are on the left and the constants are on the right. Divide both sides by 8.	$\frac{8x}{8} = \frac{24}{8}$
Simplify.	$x = 3$
Check: Substitute $x = 3$.	
$7x = -x + 24$ $7(3) \overset{?}{=} -(3) + 24$ $21 = 21$ ✓	

Note:
Exercise:

Problem: Solve: $12j = -4j + 32$.

Solution:

$j = 2$

Note:
Exercise:

Problem: Solve: $8h = -4h + 12.$

Solution:

$h = 1$

Solve Equations with Variables and Constants on Both Sides

The next example will be the first to have variables *and* constants on both sides of the equation. As we did before, we'll collect the variable terms to one side and the constants to the other side.

Example:
Exercise:

Problem: Solve: $7x + 5 = 6x + 2.$

Solution:
Solution

Start by choosing which side will be the variable side and which side will be the constant side. The variable terms are $7x$ and $6x.$ Since 7 is greater than $6,$ make the left side the variable side and so the right side will be the constant side.

	$7x + 5 = 6x + 2$
Collect the variable terms to the left side by subtracting $6x$ from both sides.	$7x - 6x + 5 = 6x - 6x + 2$
Simplify.	$x + 5 = 2$
Now, collect the constants to the right side by subtracting 5 from both sides.	$x + 5 - 5 = 2 - 5$
Simplify.	$x = -3$
The solution is $x = -3$.	
Check: Let $x = -3$.	
$7x + 5 = 6x + 2$ $7(-3) + 5 \overset{?}{=} 6(-3) + 2$ $-21 + 5 \overset{?}{=} -18 + 2$ $-16 = -16 \checkmark$	

Note:
Exercise:

Problem: Solve: $12x + 8 = 6x + 2$.

Solution:

$x = -1$

Note:
Exercise:

Problem: Solve: $9y + 4 = 7y + 12$.

Solution:

$y = 4$

We’ll summarize the steps we took so you can easily refer to them.

Note:
Solve an equation with variables and constants on both sides.

Choose one side to be the variable side and then the other will be the constant side.
Collect the variable terms to the variable side, using the Addition or Subtraction Property of Equality.
Collect the constants to the other side, using the Addition or Subtraction Property of Equality.
Make the coefficient of the variable 1, using the Multiplication or Division Property of Equality.
Check the solution by substituting it into the original equation.

It is a good idea to make the variable side the one in which the variable has the larger coefficient. This usually makes the arithmetic easier.

Example:
Exercise:

Problem: Solve: $6n - 2 = -3n + 7$.

Solution:
Solution

We have $6n$ on the left and $-3n$ on the right. Since $6 > -3$, make the left side the "variable" side.

	$6n - 2 = -3n + 7$
We don't want variables on the right side—add $3n$ to both sides to leave only constants on the right.	$6n + 3n - 2 = -3n + 3n + 7$
Combine like terms.	$9n - 2 = 7$
We don't want any constants on the left side, so add 2 to both sides.	$9n - 2 + 2 = 7 + 2$
Simplify.	$9n = 9$

The variable term is on the left and the constant term is on the right. To get the coefficient of n to be one, divide both sides by 9.	$\frac{9n}{9} = \frac{9}{9}$
Simplify.	$n = 1$
Check: Substitute 1 for n.	
$\begin{aligned} 6n - 2 &= -3n + 7 \\ 6(1) - 2 &\overset{?}{=} -3(1) + 7 \\ 4 &= 4 \checkmark \end{aligned}$	

Note:
Exercise:

Problem: Solve: $8q - 5 = -4q + 7$.

Solution:

$q = 1$

Note:
Exercise:

Problem: Solve: $7n - 3 = n + 3$.

Solution:

$n = 1$

Example:
Exercise:

Problem: Solve: $2a - 7 = 5a + 8$.

Solution:
Solution

This equation has $2a$ on the left and $5a$ on the right. Since $5 > 2$, make the right side the variable side and the left side the constant side.

	$2a - 7 = 5a + 8$
Subtract $2a$ from both sides to remove the variable term from the left.	$2a - 2a - 7 = 5a - 2a + 8$
Combine like terms.	$-7 = 3a + 8$
Subtract 8 from both sides to remove the constant from the right.	$-7 - 8 = 3a + 8 - 8$
Simplify.	$-15 = 3a$

Divide both sides by 3 to make 1 the coefficient of a.	$\frac{-15}{3} = \frac{3a}{3}$
Simplify.	$-5 = a$
Check: Let $a = -5$.	
$\begin{aligned} 2a - 7 &= 5a + 8 \\ 2(-5) - 7 &\overset{?}{=} 5(-5) + 8 \\ -10 - 7 &\overset{?}{=} -25 + 8 \\ -17 &= -17 \checkmark \end{aligned}$	

Note that we could have made the left side the variable side instead of the right side, but it would have led to a negative coefficient on the variable term. While we could work with the negative, there is less chance of error when working with positives. The strategy outlined above helps avoid the negatives!

Note:
Exercise:

Problem: Solve: $2a - 2 = 6a + 18$.

Solution:

$a = -5$

Note:
Exercise:

Problem: Solve: $4k - 1 = 7k + 17$.

Solution:

$k = -6$

To solve an equation with fractions, we still follow the same steps to get the solution.

Example:
Exercise:

Problem: Solve: $\frac{3}{2}x + 5 = \frac{1}{2}x - 3$.

Solution:
Solution

Since $\frac{3}{2} > \frac{1}{2}$, make the left side the variable side and the right side the constant side.

	$\frac{3}{2}x + 5 = \frac{1}{2}x - 3$

Subtract $\frac{1}{2}x$ from both sides.	$\frac{3}{2}x - \frac{1}{2}x + 5 = \frac{1}{2}x - \frac{1}{2}x - 3$
Combine like terms.	$x + 5 = -3$
Subtract 5 from both sides.	$x + 5 - 5 = -3 - 5$
Simplify.	$x = -8$
Check: Let $x = -8$.	
$\frac{3}{2}x + 5 = \frac{1}{2}x - 3$ $\frac{3}{2}(-8) + 5 \overset{?}{=} \frac{1}{2}(-8) - 3$ $-12 + 5 \overset{?}{=} -4 - 3$ $-7 = -7$ ✓	

Note:
Exercise:

Problem: Solve: $\frac{7}{8}x - 12 = -\frac{1}{8}x - 2$.

Solution:

$x = 10$

Note:
Exercise:

Problem: Solve: $\frac{7}{6}y + 11 = \frac{1}{6}y + 8$.

Solution:

$y = -3$

We follow the same steps when the equation has decimals, too.

Example:
Exercise:

Problem: Solve: $3.4x + 4 = 1.6x - 5$.

Solution:
Solution

Since $3.4 > 1.6$, make the left side the variable side and the right side the constant side.

	$3.4x + 4 = 1.6x - 5$

Subtract $1.6x$ from both sides.	$3.4x - 1.6x + 4 = 1.6x - 1.6x - 5$
Combine like terms.	$1.8x + 4 = -5$
Subtract 4 from both sides.	$1.8x + 4 - 4 = -5 - 4$
Simplify.	$1.8x = -9$
Use the Division Property of Equality.	$\frac{1.8x}{1.8} = \frac{-9}{1.8}$
Simplify.	$x = -5$
Check: Let $x = -5$.	
$3.4x + 4 = 1.6x - 5$ $3.4(-5) + 4 \overset{?}{=} 1.6(-5) - 5$ $-17 + 4 \overset{?}{=} -8 - 5$ $-13 = -13$ ✓	

Note:
Exercise:

Problem: Solve: $2.8x + 12 = -1.4x - 9$.

Solution:

$x = -5$

Note:
Exercise:

Problem: Solve: $3.6y + 8 = 1.2y - 4$.

Solution:

$y = -5$

Solve Equations Using a General Strategy

Each of the first few sections of this chapter has dealt with solving one specific form of a linear equation. It’s time now to lay out an overall strategy that can be used to solve *any* linear equation. We call this the *general strategy*. Some equations won’t require all the steps to solve, but many will. Simplifying each side of the equation as much as possible first makes the rest of the steps easier.

Note:
Use a general strategy for solving linear equations.

Simplify each side of the equation as much as possible. Use the Distributive Property to remove any parentheses. Combine like terms. Collect all the variable terms to one side of the equation. Use the Addition

or Subtraction Property of Equality.
Collect all the constant terms to the other side of the equation. Use the Addition or Subtraction Property of Equality.
Make the coefficient of the variable term to equal to
1. Use the Multiplication or Division Property of Equality. State the solution to the equation.
Check the solution. Substitute the solution into the original equation to make sure the result is a true statement.

Example:
Exercise:

Problem: Solve: $3(x+2)=18$.

Solution:
Solution

	$3(x+2)=18$
Simplify each side of the equation as much as possible. Use the Distributive Property.	$3x+6=18$
Collect all variable terms on one side of the equation—all xs are already on the left side.	
Collect constant terms on the other side of the equation.	$3x+6-6=18-6$

Subtract 6 from each side	
Simplify.	$3x = 12$
Make the coefficient of the variable term equal to 1. Divide each side by 3.	$\frac{3x}{3} = \frac{12}{3}$
Simplify.	$x = 4$
Check: Let $x = 4$.	
$3(x + 2) = 18$ $3(4 + 2) \overset{?}{=} 18$ $3(6) \overset{?}{=} 18$ $18 = 18$ ✓	

Note:
Exercise:

Problem: Solve: $5(x + 3) = 35$.

Solution:

$x = 4$

Note:
Exercise:

Problem: Solve: $6(y-4)=-18$.

Solution:

$y = 1$

Example:
Exercise:

Problem: Solve: $-(x+5)=7$.

Solution:
Solution

	$-(x+5)=7$
Simplify each side of the equation as much as possible by distributing. The only x term is on the left side, so all variable terms are on the left side of the equation.	$-x-5=7$
Add 5 to both sides to get all constant terms on the right side of the equation.	$-x-5+5=7+5$
Simplify.	

	$-x = 12$
Make the coefficient of the variable term equal to 1 by multiplying both sides by -1.	$-1(-x) = -1(12)$
Simplify.	$x = -12$
Check: Let $x = -12$.	
$-(x + 5) = 7$ $-(-12 + 5) \overset{?}{=} 7$ $-(-7) \overset{?}{=} 7$ $7 = 7 \checkmark$	

Note:
Exercise:

Problem: Solve: $-(y + 8) = -2$.

Solution:

$y = -6$

Note:
Exercise:

Problem: Solve: $-(z + 4) = -12.$

Solution:

$z = 8$

Example:
Exercise:

Problem: Solve: $4(x - 2) + 5 = -3.$

Solution:
Solution

	$4(x - 2) + 5 = -3$
Simplify each side of the equation as much as possible. Distribute.	$4x - 8 + 5 = -3$
Combine like terms	$4x - 3 = -3$

The only x is on the left side, so all variable terms are on one side of the equation.	
Add 3 to both sides to get all constant terms on the other side of the equation.	$4x - 3 + 3 = -3 + 3$
Simplify.	$4x = 0$
Make the coefficient of the variable term equal to 1 by dividing both sides by 4.	$\frac{4x}{4} = \frac{0}{4}$
Simplify.	$x = 0$
Check: Let $x = 0$.	
$4(x - 2) + 5 = -3$ $4(0 - 2) + 5 \overset{?}{=} -3$ $4(-2) + 5 \overset{?}{=} -3$ $-8 + 5 \overset{?}{=} -3$ $-3 = -3$ ✓	

Note:
Exercise:

Problem: Solve: $2(a - 4) + 3 = -1$.

Solution:

$a = 2$

Note:
Exercise:

Problem: Solve: $7(n-3)-8=-15.$

Solution:

$n = 2$

Example:
Exercise:

Problem: Solve: $8-2(3y+5)=0.$

Solution:
Solution

Be careful when distributing the negative.

	$8 - 2(3y + 5) = 0$

Simplify—use the Distributive Property.	$8-6y-10=0$
Combine like terms.	$-6y-2=0$
Add 2 to both sides to collect constants on the right.	$-6y-2+2=0+2$
Simplify.	$-6y=2$
Divide both sides by −6.	$\frac{-6y}{-6}=\frac{2}{-6}$
Simplify.	$y=-\frac{1}{3}$
Check: Let $y=-\frac{1}{3}$.	
$8-2(3y+5)=0$ $8-2\left[3\left(-\frac{1}{3}\right)+5\right]=0$ $8-2(-1+5)\overset{?}{=}0$ $8-2(4)\overset{?}{=}0$ $8-8\overset{?}{=}0$ $0=0\checkmark$	

Note:
Exercise:

Problem: Solve: $12 - 3(4j + 3) = -17$.

Solution:

$j = \frac{5}{3}$

Note:
Exercise:

Problem: Solve: $-6 - 8(k - 2) = -10$.

Solution:

$k = \frac{5}{2}$

Example:
Exercise:

Problem: Solve: $3(x - 2) - 5 = 4(2x + 1) + 5$.

Solution:
Solution

	$3(x-2)-5=4(2x+1)+5$
Distribute.	$3x-6-5=8x+4+5$
Combine like terms.	$3x-11=8x+9$
Subtract $3x$ to get all the variables on the right since $8>3$.	$3x-3x-11=8x-3x+9$
Simplify.	$-11=5x+9$
Subtract 9 to get the constants on the left.	$-11-9=5x+9-9$
Simplify.	$-20=5x$
Divide by 5.	$\frac{-20}{5}=\frac{5x}{5}$
Simplify.	$-4=x$
Check: Substitute: $-4=x$.	

$$\begin{aligned}3(x-2)-5&=4(2x+1)+5\\3(-4-2)-5&\overset{?}{=}4[2(-4)+1]+5\\3(-6)-5&\overset{?}{=}4(-8+1)+5\\-18-5&\overset{?}{=}4(-7)+5\\-23&\overset{?}{=}-28+5\\-23&=-23\ \checkmark\end{aligned}$$

Note:
Exercise:

Problem: Solve: $6(p-3)-7=5(4p+3)-12$.

Solution:

$p=-2$

Note:
Exercise:

Problem: Solve: $8(q+1)-5=3(2q-4)-1$.

Solution:

$q=-8$

Example:

Exercise:

Problem: Solve: $\frac{1}{2}(6x-2)=5-x$.

Solution:
Solution

	$\frac{1}{2}(6x-2)=5-x$
Distribute.	$3x-1=5-x$
Add x to get all the variables on the left.	$3x-1+x=5-x+x$
Simplify.	$4x-1=5$
Add 1 to get constants on the right.	$4x-1+1=5+1$
Simplify.	$4x=6$
Divide by 4.	$\frac{4x}{4}=\frac{6}{4}$

Simplify.	$x = \frac{3}{2}$
Check: Let $x = \frac{3}{2}$.	
$\begin{aligned} \frac{1}{2}(6x-2) &= 5-x \\ \frac{1}{2}\left(6\cdot\frac{3}{2}-2\right) &\overset{?}{=} 5-\frac{3}{2} \\ \frac{1}{2}(9-2) &\overset{?}{=} \frac{10}{2}-\frac{3}{2} \\ \frac{1}{2}(7) &\overset{?}{=} \frac{7}{2} \\ \frac{7}{2} &= \frac{7}{2}\checkmark \end{aligned}$	

Note:
Exercise:

Problem: Solve: $\frac{1}{3}(6u+3) = 7-u$.

Solution:

$u = 2$

Note:
Exercise:

Problem: Solve: $\frac{2}{3}(9x-12)=8+2x$.

Solution:

$x = 4$

In many applications, we will have to solve equations with decimals. The same general strategy will work for these equations.

Example:
Exercise:

Problem: Solve: $0.24(100x+5)=0.4(30x+15)$.

Solution:
Solution

	$0.24(100x+5)=0.4(30x+15)$
Distribute.	$24x+1.2=12x+6$
Subtract $12x$ to get all the xs to the left.	$24x+1.2-12x=12x+6-12x$

Simplify.	$12x + 1.2 = 6$
Subtract 1.2 to get the constants to the right.	$12x + 1.2 - 1.2 = 6 - 1.2$
Simplify.	$12x = 4.8$
Divide.	$\frac{12x}{12} = \frac{4.8}{12}$
Simplify.	$x = 0.4$
Check: Let $x = 0.4$.	
$\begin{aligned} 0.24(100x + 5) &= 0.4(30x + 15) \\ 0.24(100(0.4) + 5) &\overset{?}{=} 0.4(30(0.4) + 15) \\ 0.24(40 + 5) &\overset{?}{=} 0.4(12 + 15) \\ 0.24(45) &\overset{?}{=} 0.4(27) \\ 10.8 &= 10.8\ ✓ \end{aligned}$	

Note:
Exercise:

Problem: Solve: $0.55(100n + 8) = 0.6(85n + 14)$.

Solution:

1

Note:
Exercise:

Problem: Solve: $0.15(40m - 120) = 0.5(60m + 12)$.

Solution:

−1

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Solving Multi-Step Equations
- Solve an Equation with Variable Terms on Both Sides
- Solving Multi-Step Equations (L5.4)
- Solve an Equation with Variables and Parentheses on Both Sides

Key Concepts

- **Solve an equation with variables and constants on both sides**

  Choose one side to be the variable side and then the other will be the constant side.
  Collect the variable terms to the variable side, using the Addition or Subtraction Property of Equality.
  Collect the constants to the other side, using the Addition or Subtraction Property of Equality.
  Make the coefficient of the variable 1, using the Multiplication or

Division Property of Equality.
Check the solution by substituting into the original equation.

- **General strategy for solving linear equations**

  Simplify each side of the equation as much as possible. Use the Distributive Property to remove any parentheses. Combine like terms.
  Collect all the variable terms to one side of the equation. Use the Addition or Subtraction Property of Equality.
  Collect all the constant terms to the other side of the equation. Use the Addition or Subtraction Property of Equality.
  Make the coefficient of the variable term to equal to 1. Use the Multiplication or Division Property of Equality. State the solution to the equation.
  Check the solution. Substitute the solution into the original equation to make sure the result is a true statement.

Practice Makes Perfect

Solve an Equation with Constants on Both Sides

In the following exercises, solve the equation for the variable.

Exercise:

Problem: $6x - 2 = 40$

Exercise:

Problem: $7x - 8 = 34$

Solution:

6

Exercise:

Problem: $11w + 6 = 93$

Exercise:

Problem: $14y + 7 = 91$

Solution:

6

Exercise:

Problem: $3a + 8 = -46$

Exercise:

Problem: $4m + 9 = -23$

Solution:

-8

Exercise:

Problem: $-50 = 7n - 1$

Exercise:

Problem: $-47 = 6b + 1$

Solution:

-8

Exercise:

Problem: $25 = -9y + 7$

Exercise:

Problem: $29 = -8x - 3$

Solution:

-4

Exercise:

Problem: $-12p - 3 = 15$

Exercise:

Problem: $-14\mathrm{q} - 15 = 13$

Solution:

-2

Solve an Equation with Variables on Both Sides

In the following exercises, solve the equation for the variable.

Exercise:

Problem: $8z = 7z - 7$

Exercise:

Problem: $9k = 8k - 11$

Solution:

-11

Exercise:

Problem: $4x + 36 = 10x$

Exercise:

Problem: $6x + 27 = 9x$

Solution:

9

Exercise:

Problem: $c = -3c - 20$

Exercise:

Problem: $b = -4b - 15$

Solution:

−3

Exercise:

Problem: $5q = 44 - 6q$

Exercise:

Problem: $7z = 39 - 6z$

Solution:

3

Exercise:

Problem: $3y + \frac{1}{2} = 2y$

Exercise:

Problem: $8x+\frac{3}{4}=7x$

Solution:

−3/4

Exercise:

Problem: $-12a-8=-16a$

Exercise:

Problem: $-15r-8=-11r$

Solution:

−2

Solve an Equation with Variables and Constants on Both Sides

In the following exercises, solve the equations for the variable.

Exercise:

Problem: $6x-15=5x+3$

Exercise:

Problem: $4x-17=3x+2$

Solution:

19

Exercise:

Problem: $26+8d=9d+11$

Exercise:

Problem: $21+6f=7f+14$

Solution:

7

Exercise:

Problem: $3p-1=5p-33$

Exercise:

Problem: $8q-5=5q-20$

Solution:

-5

Exercise:

Problem: $4a+5=-a-40$

Exercise:

Problem: $9c+7=-2c-37$

Solution:

-4

Exercise:

Problem: $8y-30=-2y+30$

Exercise:

Problem: $12x-17=-3x+13$

Solution:

2

Exercise:

Problem: $2\mathrm{z} - 4 = 23 - \mathrm{z}$

Exercise:

Problem: $3y - 4 = 12 - y$

Solution:

4

Exercise:

Problem: $\frac{5}{4}c - 3 = \frac{1}{4}c - 16$

Exercise:

Problem: $\frac{4}{3}m - 7 = \frac{1}{3}m - 13$

Solution:

-6

Exercise:

Problem: $8 - \frac{2}{5}q = \frac{3}{5}q + 6$

Exercise:

Problem: $11 - \frac{1}{4}a = \frac{3}{4}a + 4$

Solution:

7

Exercise:

Problem: $\frac{4}{3}n + 9 = \frac{1}{3}n - 9$

Exercise:

Problem: $\frac{5}{4}a + 15 = \frac{3}{4}a - 5$

Solution:

−40

Exercise:

Problem: $\frac{1}{4}y + 7 = \frac{3}{4}y - 3$

Exercise:

Problem: $\frac{3}{5}p + 2 = \frac{4}{5}p - 1$

Solution:

15

Exercise:

Problem: $14n + 8.25 = 9n + 19.60$

Exercise:

Problem: $13z + 6.45 = 8z + 23.75$

Solution:

3.46

Exercise:

Problem: $2.4w - 100 = 0.8w + 28$

Exercise:

Problem: $2.7w - 80 = 1.2w + 10$

Solution:

60

Exercise:

Problem: $5.6r + 13.1 = 3.5r + 57.2$

Exercise:

Problem: $6.6x - 18.9 = 3.4x + 54.7$

Solution:

23

Solve an Equation Using the General Strategy

In the following exercises, solve the linear equation using the general strategy.

Exercise:

Problem: $5(x + 3) = 75$

Exercise:

Problem: $4(y + 7) = 64$

Solution:

9

Exercise:

Problem: $8 = 4(x - 3)$

Exercise:

Problem: $9 = 3(x - 3)$

Solution:

6

Exercise:

Problem: $20(y - 8) = -60$

Exercise:

Problem: $14(y - 6) = -42$

Solution:

3

Exercise:

Problem: $-4(2n + 1) = 16$

Exercise:

Problem: $-7(3n + 4) = 14$

Solution:

-2

Exercise:

Problem: $3(10+5r)=0$

Exercise:

Problem: $8(3+3\text{p})=0$

Solution:

-1

Exercise:

Problem: $\frac{2}{3}(9c-3)=22$

Exercise:

Problem: $\frac{3}{5}(10x-5)=27$

Solution:

5

Exercise:

Problem: $5(1.2u-4.8)=-12$

Exercise:

Problem: $4(2.5v-0.6)=7.6$

Solution:

0.52

Exercise:

Problem: $0.2(30n+50)=28$

Exercise:

Problem: $0.5(16m + 34) = -15$

Solution:

0.25

Exercise:

Problem: $-(w - 6) = 24$

Exercise:

Problem: $-(t - 8) = 17$

Solution:

−9

Exercise:

Problem: $9(3a + 5) + 9 = 54$

Exercise:

Problem: $8(6b - 7) + 23 = 63$

Solution:

2

Exercise:

Problem: $10 + 3(z + 4) = 19$

Exercise:

Problem: $13+2(m-4)=17$

Solution:

6

Exercise:

Problem: $7+5(4-q)=12$

Exercise:

Problem: $-9+6(5-k)=12$

Solution:

3/2

Exercise:

Problem: $15-(3r+8)=28$

Exercise:

Problem: $18-(9r+7)=-16$

Solution:

3

Exercise:

Problem: $11-4(y-8)=43$

Exercise:

Problem: $18-2(y-3)=32$

Solution:

-4

Exercise:

Problem: $9(p-1)=6(2p-1)$

Exercise:

Problem: $3(4n-1)-2=8n+3$

Solution:

2

Exercise:

Problem: $9(2m-3)-8=4m+7$

Exercise:

Problem: $5(x-4)-4x=14$

Solution:

34

Exercise:

Problem: $8(x-4)-7x=14$

Exercise:

Problem: $5+6(3s-5)=-3+2(8s-1)$

Solution:

10

Exercise:

Problem: $-12 + 8(x - 5) = -4 + 3(5x - 2)$

Exercise:

Problem: $4(x - 1) - 8 = 6(3x - 2) - 7$

Solution:

2

Exercise:

Problem: $7(2x - 5) = 8(4x - 1) - 9$

Everyday Math

Exercise:

Problem:

Making a fence Jovani has a fence around the rectangular garden in his backyard. The perimeter of the fence is 150 feet. The length is 15 feet more than the width. Find the width, w, by solving the equation $150 = 2(w + 15) + 2w$.

Solution:

30 feet

Exercise:

Problem:

Concert tickets At a school concert, the total value of tickets sold was $\$1{,}506$. Student tickets sold for $\$6$ and adult tickets sold for $\$9$. The number of adult tickets sold was 5 less than 3 times the number of student tickets. Find the number of student tickets sold, s, by solving the equation $6s+9(3s-5)=1506$.

Exercise:

Problem:

Coins Rhonda has $\$1.90$ in nickels and dimes. The number of dimes is one less than twice the number of nickels. Find the number of nickels, n, by solving the equation $0.05n+0.10(2n-1)=1.90$.

Solution:

8 nickels

Exercise:

Problem:

Fencing Micah has 74 feet of fencing to make a rectangular dog pen in his yard. He wants the length to be 25 feet more than the width. Find the length, L, by solving the equation $2L+2(L-25)=74$.

Writing Exercises

Exercise:

Problem:

When solving an equation with variables on both sides, why is it usually better to choose the side with the larger coefficient as the variable side?

Solution:

Answers will vary.

Exercise:

Problem:

Solve the equation $10x + 14 = -2x + 38$, explaining all the steps of your solution.

Exercise:

Problem:

What is the first step you take when solving the equation $3 - 7(y - 4) = 38$? Explain why this is your first step.

Solution:

Answers will vary.

Exercise:

Problem:

Solve the equation $\frac{1}{4}(8x + 20) = 3x - 4$ explaining all the steps of your solution as in the examples in this section.

Exercise:

Problem:

Using your own words, list the steps in the General Strategy for Solving Linear Equations.

Solution:

Answers will vary.

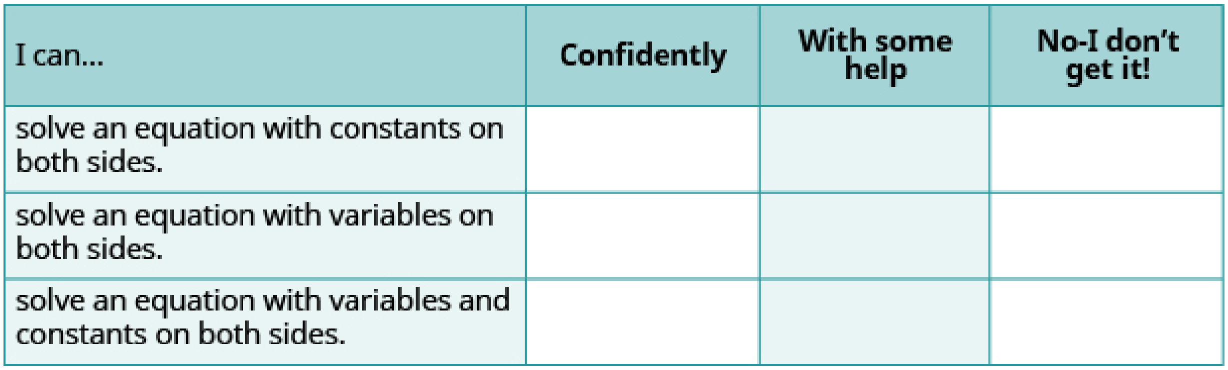
Exercise:

Problem:

Explain why you should simplify both sides of an equation as much as possible before collecting the variable terms to one side and the constant terms to the other side.

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can...	Confidently	With some help	No-I don't get it!
solve an equation with constants on both sides.			
solve an equation with variables on both sides.			
solve an equation with variables and constants on both sides.			

ⓑ What does this checklist tell you about your mastery of this section? What steps will you take to improve?

Solve Equations with Fraction or Decimal Coefficients

By the end of this section, you will be able to:

- Solve equations with fraction coefficients
- Solve equations with decimal coefficients

Note:
Before you get started, take this readiness quiz.

1. Multiply: $8 \cdot \frac{3}{8}$.
   If you missed this problem, review [link]
2. Find the LCD of $\frac{5}{6}$ and $\frac{1}{4}$.
   If you missed this problem, review [link]
3. Multiply: 4.78 by 100.
   If you missed this problem, review [link]

Solve Equations with Fraction Coefficients

Let’s use the General Strategy for Solving Linear Equations introduced earlier to solve the equation $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

	$\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$
To isolate the x term, subtract $\frac{1}{2}$ from both sides.	$\frac{1}{8}x + \frac{1}{2} - \frac{1}{2} = \frac{1}{4} - \frac{1}{2}$

Simplify the left side.	$\frac{1}{8}x = \frac{1}{4} - \frac{1}{2}$
Change the constants to equivalent fractions with the LCD.	$\frac{1}{8}x = \frac{1}{4} - \frac{2}{4}$
Subtract.	$\frac{1}{8}x = -\frac{1}{4}$
Multiply both sides by the reciprocal of $\frac{1}{8}$.	$\frac{8}{1} \cdot \frac{1}{8}x = \frac{8}{1}\left(-\frac{1}{4}\right)$
Simplify.	$x = -2$

This method worked fine, but many students don’t feel very confident when they see all those fractions. So we are going to show an alternate method to solve equations with fractions. This alternate method eliminates the fractions.

We will apply the Multiplication Property of Equality and multiply both sides of an equation by the least common denominator of *all* the fractions in the equation. The result of this operation will be a new equation, equivalent to the first, but with no fractions. This process is called *clearing the equation of fractions*. Let’s solve the same equation again, but this time use the method that clears the fractions.

Example:
Exercise:

Problem: Solve: $\frac{1}{8}x + \frac{1}{2} = \frac{1}{4}$.

Solution:
Solution

Find the least common denominator of *all* the fractions in the equation.	$\frac{1}{8}x + \frac{1}{2} = \frac{1}{4} \quad \text{LCD} = 8$
Multiply both sides of the equation by that LCD, 8. This clears the fractions.	$8\left(\frac{1}{8}x + \frac{1}{2}\right) = 8\left(\frac{1}{4}\right)$
Use the Distributive Property.	$8 \cdot \frac{1}{8}x + 8 \cdot \frac{1}{2} = 8 \cdot \frac{1}{4}$
Simplify — and notice, no more fractions!	$x + 4 = 2$
Solve using the General Strategy for Solving Linear Equations.	$x + 4 - 4 = 2 - 4$
Simplify.	$x = -2$
Check: Let $x = -2$	

$$\begin{aligned}
\frac{1}{8}x+\frac{1}{2}&=\frac{1}{4}\\
\frac{1}{8}(-2)+\frac{1}{2}&\stackrel{?}{=}\frac{1}{4}\\
-\frac{2}{8}+\frac{1}{2}&\stackrel{?}{=}\frac{1}{4}\\
-\frac{2}{8}+\frac{4}{8}&\stackrel{?}{=}\frac{1}{4}\\
\frac{2}{4}&\stackrel{?}{=}\frac{1}{4}\\
\frac{1}{4}&=\frac{1}{4}\checkmark
\end{aligned}$$

Note:
Exercise:

Problem: Solve: $\frac{1}{4}x+\frac{1}{2}=\frac{5}{8}$.

Solution:

$x=\frac{1}{2}$

Note:
Exercise:

Problem: Solve: $\frac{1}{6}y-\frac{1}{3}=\frac{1}{6}$.

Solution:

$y = 3$

Notice in [link] that once we cleared the equation of fractions, the equation was like those we solved earlier in this chapter. We changed the problem to one we already knew how to solve! We then used the General Strategy for Solving Linear Equations.

Note:
Solve equations with fraction coefficients by clearing the fractions.

Find the least common denominator of *all* the fractions in the equation.
Multiply both sides of the equation by that LCD. This clears the fractions.
Solve using the General Strategy for Solving Linear Equations.

Example:
Exercise:

Problem: Solve: $7 = \frac{1}{2}x + \frac{3}{4}x - \frac{2}{3}x.$

Solution:
Solution

We want to clear the fractions by multiplying both sides of the equation by the LCD of all the fractions in the equation.

Find the least common denominator of *all* the fractions in the equation.	$7=\frac{1}{2}x+\frac{3}{4}x-\frac{2}{3}x$ LCD = 12
Multiply both sides of the equation by 12.	$12(7)=12\cdot\frac{1}{2}x+\frac{3}{4}x-\frac{2}{3}x$
Distribute.	$12(7)=12\cdot\frac{1}{2}x+12\cdot\frac{3}{4}x-12\cdot\frac{2}{3}x$
Simplify — and notice, no more fractions!	$84=6x+9x-8x$
Combine like terms.	$84=7x$
Divide by 7.	$\frac{84}{7}=\frac{7x}{7}$
Simplify.	$12=x$
Check: Let $x=12$.	
$7=\frac{1}{2}x+\frac{3}{4}x-\frac{2}{3}x$ $7\overset{?}{=}\frac{1}{2}(12)+\frac{3}{4}(12)-\frac{2}{3}(12)$ $7\overset{?}{=}6+9-8$ $7=7$ ✓	

Note:
Exercise:

Problem: Solve: $6=\frac{1}{2}v+\frac{2}{5}v-\frac{3}{4}v.$

Solution:

$v=40$

Note:
Exercise:

Problem: Solve: $-1=\frac{1}{2}u+\frac{1}{4}u-\frac{2}{3}u.$

Solution:

$u=-12$

In the next example, we’ll have variables and fractions on both sides of the equation.

Example:
Exercise:

Problem: Solve: $x+\frac{1}{3}=\frac{1}{6}x-\frac{1}{2}.$

Solution:
Solution

Find the LCD of all the fractions in the equation.	$x+\frac{1}{3}=\frac{1}{6}x-\frac{1}{2}$, LCD $=6$
Multiply both sides by the LCD.	$6\left(x+\frac{1}{3}\right)=6\left(\frac{1}{6}x-\frac{1}{2}\right)$
Distribute.	$6\cdot x+6\cdot\frac{1}{3}=6\cdot\frac{1}{6}x-6\cdot\frac{1}{2}$
Simplify — no more fractions!	$6x+2=x-3$
Subtract x from both sides.	$6x-x+2=x-x-3$
Simplify.	$5x+2=-3$
Subtract 2 from both sides.	$5x+2-2=-3-2$
Simplify.	$5x=-5$
Divide by 5.	$\frac{5x}{5}=\frac{-5}{5}$
Simplify.	$x=-1$

Check: Substitute $x = -1$.

$$
\begin{aligned}
x + \frac{1}{3} &= \frac{1}{6}x - \frac{1}{2} \\
(-1) + \frac{1}{3} &\overset{?}{=} \frac{1}{6}(-1) - \frac{1}{2} \\
(-1) + \frac{1}{3} &\overset{?}{=} -\frac{1}{6} - \frac{1}{2} \\
-\frac{3}{3} + \frac{1}{3} &\overset{?}{=} -\frac{1}{6} - \frac{3}{6} \\
-\frac{2}{3} &\overset{?}{=} -\frac{4}{6} \\
-\frac{2}{3} &= -\frac{2}{3} \checkmark
\end{aligned}
$$

Note:
Exercise:

Problem: Solve: $a + \frac{3}{4} = \frac{3}{8}a - \frac{1}{2}$.

Solution:

$a = -2$

Note:
Exercise:

Problem: Solve: $c + \frac{3}{4} = \frac{1}{2}c - \frac{1}{4}$.

Solution:

$c = -2$

In [link], we’ll start by using the Distributive Property. This step will clear the fractions right away!

Example:
Exercise:

Problem: Solve: $1 = \frac{1}{2}(4x + 2)$.

Solution:
Solution

	$1 = \frac{1}{2}(4x + 2)$
Distribute.	$1 = \frac{1}{2} \cdot 4x + \frac{1}{2} \cdot 2$
Simplify. Now there are no fractions to clear!	$1 = 2x + 1$
Subtract 1 from both sides.	

	$1 - 1 = 2x + 1 - 1$
Simplify.	$0 = 2x$
Divide by 2.	$\frac{0}{2} = \frac{2x}{2}$
Simplify.	$0 = x$
Check: Let $x = 0$.	
$1 = \frac{1}{2}(4x + 2)$ $1 \stackrel{?}{=} \frac{1}{2}(4(0) + 2)$ $1 \stackrel{?}{=} \frac{1}{2}(2)$ $1 \stackrel{?}{=} \frac{2}{2}$ $1 = 1$ ✓	

Note:
Exercise:

Problem: Solve: $-11 = \frac{1}{2}(6p + 2)$.

Solution:

$p = -4$

Note:
Exercise:

Problem: Solve: $8 = \frac{1}{3}(9q + 6)$.

Solution:

$q = 2$

Many times, there will still be fractions, even after distributing.

Example:
Exercise:

Problem: Solve: $\frac{1}{2}(y - 5) = \frac{1}{4}(y - 1)$.

Solution:
Solution

	$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$
Distribute.	$\frac{1}{2}\cdot y-\frac{1}{2}\cdot 5=\frac{1}{4}\cdot y-\frac{1}{4}\cdot 1$
Simplify.	$\frac{1}{2}y-\frac{5}{2}=\frac{1}{4}y-\frac{1}{4}$
Multiply by the LCD, 4.	$4\left(\frac{1}{2}y-\frac{5}{2}\right)=4\left(\frac{1}{4}y-\frac{1}{4}\right)$
Distribute.	$4\cdot\frac{1}{2}y-4\cdot\frac{5}{2}=4\cdot\frac{1}{4}y-4\cdot\frac{1}{4}$
Simplify.	$2y-10=y-1$
Collect the y terms to the left.	$2y-10-y=y-1-y$
Simplify.	$y-10=-1$
Collect the constants to the right.	$y-10+10=-1+10$
Simplify.	$y=9$

Check: Substitute 9 for y.	
$\frac{1}{2}(y-5)=\frac{1}{4}(y-1)$ $\frac{1}{2}(9-5)\overset{?}{=}\frac{1}{4}(9-1)$ $\frac{1}{2}(4)\overset{?}{=}\frac{1}{4}(8)$ $2=2$ ✓	

Note:
Exercise:

Problem: Solve: $\frac{1}{5}(n+3)=\frac{1}{4}(n+2)$.

Solution:

$n = 2$

Note:
Exercise:

Problem: Solve: $\frac{1}{2}(m-3)=\frac{1}{4}(m-7)$.

Solution:

$m = -1$

Solve Equations with Decimal Coefficients

Some equations have decimals in them. This kind of equation will occur when we solve problems dealing with money and percent. But decimals are really another way to represent fractions. For example, $0.3 = \frac{3}{10}$ and $0.17 = \frac{17}{100}$. So, when we have an equation with decimals, we can use the same process we used to clear fractions—multiply both sides of the equation by the least common denominator.

Example:
Exercise:

Problem: Solve: $0.8x - 5 = 7$.

Solution:
Solution

The only decimal in the equation is 0.8. Since $0.8 = \frac{8}{10}$, the LCD is 10. We can multiply both sides by 10 to clear the decimal.

	$0.8x - 5 = 7$
Multiply both sides by the LCD.	$10(0.8x - 5) = 10(7)$
Distribute.	$10(0.8x) - 10(5) = 10(7)$

Multiply, and notice, no more decimals!	$8x - 50 = 70$
Add 50 to get all constants to the right.	$8x - 50 + 50 = 70 + 50$
Simplify.	$8x = 120$
Divide both sides by 8.	$\frac{8x}{8} = \frac{120}{8}$
Simplify.	$x = 15$
Check: Let $x = 15$.	
$0.8(15) - 5 \overset{?}{=} 7$ $12 - 5 \overset{?}{=} 7$ $7 = 7$ ✓	

Note:
Exercise:

Problem: Solve: $0.6x - 1 = 11$.

Solution:

$x = 20$

Note:
Exercise:

Problem: Solve: $1.2x - 3 = 9$.

Solution:

$x = 10$

Example:
Exercise:

Problem: Solve: $0.06x + 0.02 = 0.25x - 1.5$.

Solution:
Solution

Look at the decimals and think of the equivalent fractions.

$$0.06 = \frac{6}{100}, \quad 0.02 = \frac{2}{100}, \quad 0.25 = \frac{25}{100}, \quad 1.5 = 1\frac{5}{10}$$

Notice, the LCD is 100.

By multiplying by the LCD we will clear the decimals.

	$0.06x + 0.02 = 0.25x - 1.5$
Multiply both sides by 100.	$100(0.06x + 0.02) = 100(0.25x - 1.5)$
Distribute.	$100(0.06x) + 100(0.02) = 100(0.25x) - 100(1.5)$
Multiply, and now no more decimals.	$6x + 2 = 25x - 150$
Collect the variables to the right.	$6x - 6x + 2 = 25x - 6x - 150$
Simplify.	$2 = 19x - 150$
Collect the constants to the left.	$2 + 150 = 19x - 150 + 150$
Simplify.	$152 = 19x$
Divide by 19.	$\frac{152}{19} = \frac{19x}{19}$
Simplify.	$8 = x$
Check: Let $x = 8$.	

$$0.06(8) + 0.02 = 0.25(8) - 1.5$$

$$0.48 + 0.02 = 2.00 - 1.5$$

$$0.50 = 0.50 \checkmark$$

Note:
Exercise:

Problem: Solve: $0.14h + 0.12 = 0.35h - 2.4$.

Solution:

$h = 12$

Note:
Exercise:

Problem: Solve: $0.65k - 0.1 = 0.4k - 0.35$.

Solution:

$k = -1$

The next example uses an equation that is typical of the ones we will see in the money applications in the next chapter. Notice that we will distribute the decimal first before we clear all decimals in the equation.

Example:

Exercise:

Problem: Solve: $0.25x + 0.05(x + 3) = 2.85$.

Solution:
Solution

	$0.25x + 0.05(x + 3) = 2.85$
Distribute first.	$0.25x + 0.05x + 0.15 = 2.85$
Combine like terms.	$0.30x + 0.15 = 2.85$
To clear decimals, multiply by 100.	$100(0.30x + 0.15) = 100(2.85)$
Distribute.	$30x + 15 = 285$
Subtract 15 from both sides.	$30x + 15 - 15 = 285 - 15$
Simplify.	$30x = 270$

Divide by 30.	$\frac{30x}{30} = \frac{270}{30}$
Simplify.	$x = 9$
Check: Let $x = 9$.	
$\begin{aligned} 0.25x + 0.05(x+3) &= 2.85 \\ 0.25(9) + 0.05(9+3) &\overset{?}{=} 2.85 \\ 2.25 + 0.05(12) &\overset{?}{=} 2.85 \\ 2.25 + 0.60 &\overset{?}{=} 2.85 \\ 2.85 &= 2.85 \checkmark \end{aligned}$	

Note:
Exercise:

Problem: Solve: $0.25n + 0.05(n + 5) = 2.95$.

Solution:

$n = 9$

Note:
Exercise:

Problem: Solve: $0.10d + 0.05(d - 5) = 2.15$.

Solution:

$d = 16$

Note:
ACCESS ADDITIONAL ONLINE RESOURCES

- Solve an Equation with Fractions with Variable Terms on Both Sides
- Ex 1: Solve an Equation with Fractions with Variable Terms on Both Sides
- Ex 2: Solve an Equation with Fractions with Variable Terms on Both Sides
- Solving Multiple Step Equations Involving Decimals
- Ex: Solve a Linear Equation With Decimals and Variables on Both Sides
- Ex: Solve an Equation with Decimals and Parentheses

Key Concepts

- **Solve equations with fraction coefficients by clearing the fractions.**

  Find the least common denominator of *all* the fractions in the equation.
  Multiply both sides of the equation by that LCD. This clears the fractions.
  Solve using the General Strategy for Solving Linear Equations.

Section Exercises

Practice Makes Perfect

Solve equations with fraction coefficients

In the following exercises, solve the equation by clearing the fractions.

Exercise:

Problem: $\frac{1}{4}x - \frac{1}{2} = -\frac{3}{4}$

Solution:

$x = -1$

Exercise:

Problem: $\frac{3}{4}x - \frac{1}{2} = \frac{1}{4}$

Exercise:

Problem: $\frac{5}{6}y - \frac{2}{3} = -\frac{3}{2}$

Solution:

$y = -1$

Exercise:

Problem: $\frac{5}{6}y - \frac{1}{3} = -\frac{7}{6}$

Exercise:

Problem: $\frac{1}{2}a + \frac{3}{8} = \frac{3}{4}$

Solution:

$a = \frac{3}{4}$

Exercise:

Problem: $\frac{5}{8}b+\frac{1}{2}=-\frac{3}{4}$

Exercise:

Problem: $2=\frac{1}{3}x-\frac{1}{2}x+\frac{2}{3}x$

Solution:

$x = 4$

Exercise:

Problem: $2=\frac{3}{5}x-\frac{1}{3}x+\frac{2}{5}x$

Exercise:

Problem: $\frac{1}{4}m-\frac{4}{5}m+\frac{1}{2}m=-1$

Solution:

$m = 20$

Exercise:

Problem: $\frac{5}{6}n-\frac{1}{4}n-\frac{1}{2}n=-2$

Exercise:

Problem: $x+\frac{1}{2}=\frac{2}{3}x-\frac{1}{2}$

Solution:

$x = -3$

Exercise:

Problem: $x + \frac{3}{4} = \frac{1}{2}x - \frac{5}{4}$

Exercise:

Problem: $\frac{1}{3}w + \frac{5}{4} = w - \frac{1}{4}$

Solution:

$w = \frac{9}{4}$

Exercise:

Problem: $\frac{3}{2}z + \frac{1}{3} = z - \frac{2}{3}$

Exercise:

Problem: $\frac{1}{2}x - \frac{1}{4} = \frac{1}{12}x + \frac{1}{6}$

Solution:

$x = 1$

Exercise:

Problem: $\frac{1}{2}a - \frac{1}{4} = \frac{1}{6}a + \frac{1}{12}$

Exercise:

Problem: $\frac{1}{3}b + \frac{1}{5} = \frac{2}{5}b - \frac{3}{5}$

Solution:

$b = 12$

Exercise:

Problem: $\frac{1}{3}x + \frac{2}{5} = \frac{1}{5}x - \frac{2}{5}$

Exercise:

Problem: $1 = \frac{1}{6}(12x - 6)$

Solution:

$x = 1$

Exercise:

Problem: $1 = \frac{1}{5}(15x - 10)$

Exercise:

Problem: $\frac{1}{4}(p - 7) = \frac{1}{3}(p + 5)$

Solution:

$p = -41$

Exercise:

Problem: $\frac{1}{5}(q + 3) = \frac{1}{2}(q - 3)$

Exercise:

Problem: $\frac{1}{2}(x + 4) = \frac{3}{4}$

Solution:

$x = -\frac{5}{2}$

Exercise:

Problem: $\frac{1}{3}(x+5)=\frac{5}{6}$

Solve Equations with Decimal Coefficients

In the following exercises, solve the equation by clearing the decimals.

Exercise:

Problem: $0.6y+3=9$

Solution:

$y = 10$

Exercise:

Problem: $0.4y-4=2$

Exercise:

Problem: $3.6j-2=5.2$

Solution:

$j = 2$

Exercise:

Problem: $2.1k+3=7.2$

Exercise:

Problem: $0.4x+0.6=0.5x-1.2$

Solution:

$x = 18$

Exercise:

Problem: $0.7x + 0.4 = 0.6x + 2.4$

Exercise:

Problem: $0.23x + 1.47 = 0.37x - 1.05$

Solution:

$x = 18$

Exercise:

Problem: $0.48x + 1.56 = 0.58x - 0.64$

Exercise:

Problem: $0.9x - 1.25 = 0.75x + 1.75$

Solution:

$x = 20$

Exercise:

Problem: $1.2x - 0.91 = 0.8x + 2.29$

Exercise:

Problem: $0.05n + 0.10(n + 8) = 2.15$

Solution:

$n = 9$

Exercise:

Problem: $0.05n + 0.10(n + 7) = 3.55$

Exercise:

Problem: $0.10d + 0.25(d + 5) = 4.05$

Solution:

$d = 8$

Exercise:

Problem: $0.10d + 0.25(d + 7) = 5.25$

Exercise:

Problem: $0.05(q - 5) + 0.25q = 3.05$

Solution:

$q = 11$

Exercise:

Problem: $0.05(q - 8) + 0.25q = 4.10$

Everyday Math

Exercise:

Problem:

Coins Taylor has \$2.00 in dimes and pennies. The number of pennies is 2 more than the number of dimes. Solve the equation $0.10d + 0.01(d + 2) = 2$ for d, the number of dimes.

Solution:

$d = 18$

Exercise:

Problem:

Stamps Travis bought $\$9.45$ worth of 49-cent stamps and 21-cent stamps. The number of 21-cent stamps was 5 less than the number of 49-cent stamps. Solve the equation $0.49s + 0.21(s - 5) = 9.45$ for s, to find the number of 49-cent stamps Travis bought.

Writing Exercises

Exercise:

Problem:

Explain how to find the least common denominator of $\frac{3}{8}$, $\frac{1}{6}$, and $\frac{2}{3}$.

Solution:

Answers will vary.

Exercise:

Problem:

If an equation has several fractions, how does multiplying both sides by the LCD make it easier to solve?

Exercise:

Problem:

If an equation has fractions only on one side, why do you have to multiply both sides of the equation by the LCD?

Solution:

Answers will vary.

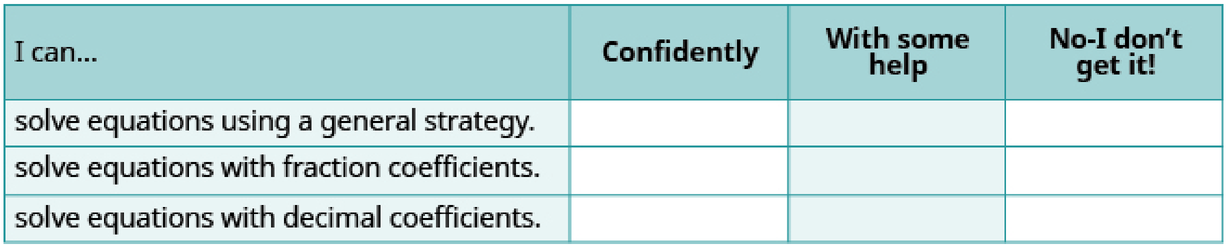
Exercise:

Problem:

In the equation $0.35x+2.1=3.85$, what is the LCD? How do you know?

Self Check

ⓐ After completing the exercises, use this checklist to evaluate your mastery of the objectives of this section.



I can…	Confidently	With some help	No-I don't get it!
solve equations using a general strategy.			
solve equations with fraction coefficients.			
solve equations with decimal coefficients.			

ⓑ Overall, after looking at the checklist, do you think you are well-prepared for the next Chapter? Why or why not?

Chapter Review Exercises

Solve Equations using the Subtraction and Addition Properties of Equality

In the following exercises, determine whether the given number is a solution to the equation.

Exercise:

Problem: $x+16=31,\ x=15$

Solution:

yes

Exercise:

Problem: $w - 8 = 5,\ w = 3$

Exercise:

Problem: $-9n = 45,\ n = 54$

Solution:

no

Exercise:

Problem: $4a = 72,\ a = 18$

In the following exercises, solve the equation using the Subtraction Property of Equality.

Exercise:

Problem: $x + 7 = 19$

Solution:

12

Exercise:

Problem: $y + 2 = -6$

Exercise:

Problem: $a + \frac{1}{3} = \frac{5}{3}$

Solution:

$a = \frac{4}{3}$

Exercise:

Problem: $n + 3.6 = 5.1$

In the following exercises, solve the equation using the Addition Property of Equality.

Exercise:

Problem: $u - 7 = 10$

Solution:

$u = 17$

Exercise:

Problem: $x - 9 = -4$

Exercise:

Problem: $c - \frac{3}{11} = \frac{9}{11}$

Solution:

$c = \frac{12}{11}$

Exercise:

Problem: $p - 4.8 = 14$

In the following exercises, solve the equation.

Exercise:

Problem: $n - 12 = 32$

Solution:

$n = 44$

Exercise:

Problem: $y + 16 = -9$

Exercise:

Problem: $f + \frac{2}{3} = 4$

Solution:

$f = \frac{10}{3}$

Exercise:

Problem: $d - 3.9 = 8.2$

Exercise:

Problem: $y + 8 - 15 = -3$

Solution:

$y = 4$

Exercise:

Problem: $7x + 10 - 6x + 3 = 5$

Exercise:

Problem: $6(n - 1) - 5n = -14$

Solution:

$n = -8$

Exercise:

Problem: $8(3p+5)-23(p-1)=35$

In the following exercises, translate each English sentence into an algebraic equation and then solve it.

Exercise:

Problem: The sum of -6 and m is 25.

Solution:

$-6 + m = 25$; $m = 31$

Exercise:

Problem: Four less than n is 13.

In the following exercises, translate into an algebraic equation and solve.

Exercise:

Problem:

Rochelle’s daughter is 11 years old. Her son is 3 years younger. How old is her son?

Solution:

$s = 11 - 3$; 8 years old

Exercise:

Problem:

Tan weighs 146 pounds. Minh weighs 15 pounds more than Tan. How much does Minh weigh?

Exercise:

Problem:

Peter paid $\$9.75$ to go to the movies, which was $\$46.25$ less than he paid to go to a concert. How much did he pay for the concert?

Solution:

$c - 46.25 = 9.75$; \$56.00

Exercise:

Problem:

Elissa earned $\$152.84$ this week, which was $\$21.65$ more than she earned last week. How much did she earn last week?

Solve Equations using the Division and Multiplication Properties of Equality

In the following exercises, solve each equation using the Division Property of Equality.

Exercise:

Problem: $8x = 72$

Solution:

$x = 9$

Exercise:

Problem: $13a = -65$

Exercise:

Problem: $0.25p = 5.25$

Solution:

$p = 21$

Exercise:

Problem: $-y = 4$

In the following exercises, solve each equation using the Multiplication Property of Equality.

Exercise:

Problem: $\frac{n}{6} = 18$

Solution:

$n = 108$

Exercise:

Problem: $\frac{y}{-10} = 30$

Exercise:

Problem: $36 = \frac{3}{4}x$

Solution:

$x = 48$

Exercise:

Problem: $\frac{5}{8}u = \frac{15}{16}$

In the following exercises, solve each equation.

Exercise:

Problem: $-18m = -72$

Solution:

$m = 4$

Exercise:

Problem: $\frac{c}{9} = 36$

Exercise:

Problem: $0.45x = 6.75$

Solution:

$x = 15$

Exercise:

Problem: $\frac{11}{12} = \frac{2}{3}y$

Exercise:

Problem: $5r - 3r + 9r = 35 - 2$

Solution:

$r = 3$

Exercise:

Problem: $24x + 8x - 11x = -7 - 14$

Solve Equations with Variables and Constants on Both Sides

In the following exercises, solve the equations with constants on both sides.
Exercise:

Problem: $8p+7=47$

Solution:

$p = 5$

Exercise:

Problem: $10w-5=65$

Exercise:

Problem: $3x+19=-47$

Solution:

$x = -22$

Exercise:

Problem: $32=-4-9n$

In the following exercises, solve the equations with variables on both sides.
Exercise:

Problem: $7y=6y-13$

Solution:

$y = -13$

Exercise:

Problem: $5a+21=2a$

Exercise:

Problem: $k = -6k - 35$

Solution:

$k = -5$

Exercise:

Problem: $4x - \frac{3}{8} = 3x$

In the following exercises, solve the equations with constants and variables on both sides.

Exercise:

Problem: $12x - 9 = 3x + 45$

Solution:

$x = 6$

Exercise:

Problem: $5n - 20 = -7n - 80$

Exercise:

Problem: $4u + 16 = -19 - u$

Solution:

$u = -7$

Exercise:

Problem: $\frac{5}{8}c - 4 = \frac{3}{8}c + 4$

In the following exercises, solve each linear equation using the general strategy.

Exercise:

Problem: $6(x+6)=24$

Solution:

$x = -2$

Exercise:

Problem: $9(2p-5)=72$

Exercise:

Problem: $-(s+4)=18$

Solution:

$s = -22$

Exercise:

Problem: $8+3(n-9)=17$

Exercise:

Problem: $23-3(y-7)=8$

Solution:

$y = 12$

Exercise:

Problem: $\frac{1}{3}(6m+21)=m-7$

Exercise:

Problem: $8(r-2)=6(r+10)$

Solution:

$r = 38$

Exercise:

Problem: $5+7(2-5x)=2(9x+1)-(13x-57)$

Exercise:

Problem: $4(3.5y+0.25)=365$

Solution:

$y = 26$

Exercise:

Problem: $0.25(q-8)=0.1(q+7)$

Solve Equations with Fraction or Decimal Coefficients

In the following exercises, solve each equation by clearing the fractions.

Exercise:

Problem: $\frac{2}{5}n-\frac{1}{10}=\frac{7}{10}$

Solution:

$n = 2$

Exercise:

Problem: $\frac{1}{3}x + \frac{1}{5}x = 8$

Exercise:

Problem: $\frac{3}{4}a - \frac{1}{3} = \frac{1}{2}a + \frac{5}{6}$

Solution:

$a = \frac{14}{3}$

Exercise:

Problem: $\frac{1}{2}(k + 3) = \frac{1}{3}(k + 16)$

In the following exercises, solve each equation by clearing the decimals.

Exercise:

Problem: $0.8x - 0.3 = 0.7x + 0.2$

Solution:

$x = 5$

Exercise:

Problem: $0.36u + 2.55 = 0.41u + 6.8$

Exercise:

Problem: $0.6p - 1.9 = 0.78p + 1.7$

Solution:

$p = -20$

Exercise:

Problem: $0.10d + 0.05(d - 4) = 2.05$

Chapter Practice Test

Exercise:

Problem:

Determine whether each number is a solution to the equation.
$3x + 5 = 23.$

ⓐ 6
ⓑ $\frac{23}{5}$

Solution:

ⓐ yes
ⓑ no

In the following exercises, solve each equation.

Exercise:

Problem: $n - 18 = 31$

Exercise:

Problem: $9c = 144$

Solution:

$c = 16$

Exercise:

Problem: $4y - 8 = 16$

Exercise:

Problem: $-8x - 15 + 9x - 1 = -21$

Solution:

$x = -5$

Exercise:

Problem: $-15a = 120$

Exercise:

Problem: $\frac{2}{3}x = 6$

Solution:

$x = 9$

Exercise:

Problem: $x + 3.8 = 8.2$

Exercise:

Problem: $10y = -5y + 60$

Solution:

$y = 4$

Exercise:

Problem: $8n + 2 = 6n + 12$

Exercise:

Problem: $9m - 2 - 4m + m = 42 - 8$

Solution:

$m = 6$

Exercise:

Problem: $-5(2x + 1) = 45$

Exercise:

Problem: $-(d + 9) = 23$

Solution:

$d = -32$

Exercise:

Problem: $\frac{1}{3}(6m + 21) = m - 7$

Exercise:

Problem: $2(6x + 5) - 8 = -22$

Solution:

$x = -2$

Exercise:

Problem: $8(3a + 5) - 7(4a - 3) = 20 - 3a$

Exercise:

Problem: $\frac{1}{4}p + \frac{1}{3} = \frac{1}{2}$

Solution:

$p = \frac{2}{3}$

Exercise:

Problem: $0.1d + 0.25(d + 8) = 4.1$

Exercise:

Problem: Translate and solve: The difference of twice x and 4 is 16.

Solution:

$2x - 4 = 16$; $x = 10$

Exercise:

Problem:

Samuel paid $\$25.82$ for gas this week, which was $\$3.47$ less than he paid last week. How much did he pay last week?

Cumulative Review

Note: Answers to the Cumulative Review can be found in the Supplemental Resources. Please visit http://openstaxcollege.org to view an updated list of the Learning Resources for this title and how to access them.

Chapter 1 Whole Numbers

No exercises.

Chapter 2 The Language of Algebra

Simplify:

1. $5(3+2\cdot 6)-8^2$

Solve:

2. $17=y-13$

3. $p+14=23$

Translate into an algebraic expression.

4. 11 less than the product of 7 and x.

Translate into an algebraic equation and solve.

5. Twice the difference of y and 7 gives 84.

6. Find all the factors of 72.

7. Find the prime factorization of 132.

8. Find the least common multiple of 12 and 20.

Chapter 3 Integers

Simplify:

9. $|8-9|-|3-8|$

10. $-2+4(-3+7)$

11. $27-(-4-7)$

12. $28\div(-4)-7$

Translate into an algebraic expression or equation.

13. The sum of -5 and 13, increased by 11.

14. The product of -11 and 8.

15. The quotient of 7 and the sum of -4 and m.

16. The product of -3 and is -51.

Solve:

17. $-6r=24$

Chapter 4 Fractions

18. Locate the numbers on a number line. $\frac{7}{8}, \frac{5}{3}, 3\frac{1}{4}, 5$.

Simplify:

19. $\frac{21p}{57q}$

20. $\frac{3}{7}\cdot\left(-\frac{28}{45}\right)$

21. $-6\frac{3}{4} \div \frac{9}{2}$

22. $-3\frac{3}{5} \div 6$

23. $-4\frac{2}{3}\left(-\frac{6}{7}\right)$

24. $\frac{-2\frac{1}{4}}{-\frac{3}{8}}$

25. $\frac{7 \cdot 8+4(7-12)}{9 \cdot 6-2 \cdot 9}$

26. $-\frac{23}{36}+\frac{17}{20}$

27. $\frac{\frac{1}{2}+\frac{1}{3}}{\frac{3}{4}-\frac{1}{3}}$

28. $3\frac{5}{8}-2\frac{1}{2}$

29. $-\frac{2}{3}r=24$

Chapter 5 Decimals

Simplify:

30. $24.76-7.28$

31. $12.9+15.633$

32. $(-5.6)(0.25)$

33. $\$6.29 \div 12$

34. $\frac{3}{4}(13.44-9.6)$

35. $\sqrt{64}+\sqrt{225}$

36. $\sqrt{121x^2y^2}$

37. Write in order from smallest to largest: $\frac{5}{8}$, 0.75, $\frac{8}{15}$

Solve:

38. $-8.6x = 34.4$

39. Using 3.14 as the estimate for pi, approximate the (a) circumference and (b) area of a circle whose radius is 8 inches.

40. Find the mean of the numbers, $18, 16, 20, 12$

41. Find the median of the numbers, $24, 29, 27, 28, 30$

42. Identify the mode of the numbers, $6, 4, 4, 5, 6, 6, 4, 4, 4, 3, 5$

43. Find the unit price of one t-shirt if they are sold at 3 for $\$28.97$.

Chapter 6 Percents

44. Convert 14.7% to (a) a fraction and (b) a decimal.

Translate and solve.

45. 63 is 35% of what number?

46. The nutrition label on a package of granola bars says that each granola bar has 180 calories, and 81 calories are from fat. What percent of the total calories is from fat?

47. Elliot received $\$510$ commission when he sold a $\$3,400$ painting at the art gallery where he works. What was the rate of commission?

48. Nandita bought a set of towels on sale for $\$67.50$. The original price of the towels was $\$90$. What was the discount rate?

49. Alan invested $23,000 in a friend's business. In 5 years the friend paid him the $23,000 plus $9,200 interest. What was the rate of interest?

Solve:

50. $\frac{9}{p} = \frac{-6}{14}$

Chapter 7 The Properties of Real Numbers

51. List the (a) whole numbers, (b) integers, (c) rational numbers, (d) irrational numbers,

(e) real numbers $-5, -2\frac{1}{4}, -\sqrt{4}, 0.25, \frac{13}{5}, 4$

Simplify:

52. $\left(\frac{8}{15} + \frac{4}{7}\right) + \frac{3}{7}$

53. $3(y+3) - 8(y-4)$

54. $\frac{8}{17} \cdot 49 \cdot \frac{17}{8}$

55. A playground is 55 feet wide. Convert the width to yards.

56. Every day last week Amit recorded the number of minutes he spent reading. The recorded number of minutes he read each day was $48, 26, 81, 54, 43, 62, 106$. How many hours did Amit spend reading last week?

57. June walked 2.8 kilometers. Convert this length to miles knowing 1 mile is 1.61 kilometer.

Chapter 8 Solve Linear Equations

Solve:

58. $y + 13 = -8$

59. $p + \frac{2}{5} = \frac{8}{5}$

60. $48 = \frac{2}{3}x$

61. $4\left(a - 3\right) - 6a = -18$

62. $7q + 14 = -35$

63. $4v - 27 = 7v$

64. $\frac{7}{8}y - 6 = \frac{3}{8}y - 8$

65. $26 - 4(z - 2) = 6$

66. $\frac{3}{4}\,x - \frac{2}{3} = \frac{1}{2}x - \frac{5}{6}$

67. $0.7y + 4.8 = 0.84y - 5.3$

Translate and solve.

68. Four less than n is 13.

Chapter 9 Math Models and Geometry

69. One number is 8 less than another. Their sum is negative twenty-two. Find the numbers.

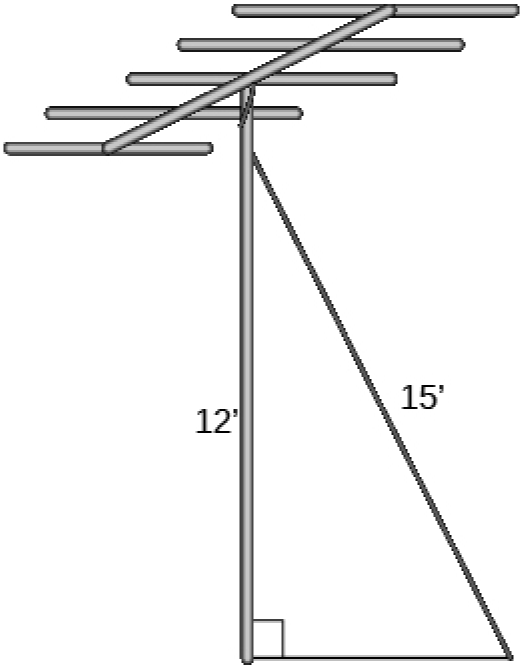
70. The sum of two consecutive integers is -95. Find the numbers.

71. Wilma has $\$3.65$ in dimes and quarters. The number of dimes is 2 less than the number of quarters. How many of each coin does she have?

72. Two angles are supplementary. The larger angle is 24° more than the smaller angle. Find the measurements of both angles.

73. One angle of a triangle is 20° more than the smallest angle. The largest angle is the sum of the other angles. Find the measurements of all three angles.

74. Erik needs to attach a wire to hold the antenna to the roof of his house, as shown in the figure. The antenna is 12 feet tall and Erik has 15 feet of wire. How far from the base of the antenna can he attach the wire?



75. The width of a rectangle is 4 less than the length. The perimeter is 96 inches. Find the length and the width.

76. Find the (a) volume and (b) surface area of a rectangular carton with length 24 inches, width 18 inches, and height 6 inches.

Chapter 10 Polynomials

Simplify:

77. $\left(8m^2+12m-5\right)-\left(2m^2-7m-1\right)$

78. $p^3\cdot p^{10}$

79. $\left(y^4\right)^3$

80. $\left(3a^5\right)^3$

81. $\left(x^3\right)^5\left(x^2\right)^3$

82. $\left(\frac{2}{3}m^3n^6\right)\left(\frac{1}{6}m^4n^4\right)$

83. $(y-4)(y+12)$

84. $(3c+1)(9c-4)$

85. $(x-1)\left(x^2-3x-2\right)$

86. $(8x)^0$

87. $\frac{\left(x^3\right)^5}{\left(x^2\right)^4}$

88. $\frac{32a^7b^2}{12a^3b^6}$

89. $\left(ab^{-3}\right)\left(a^{-3}b^6\right)$

90. Write in scientific notation: (a) 4,800,000 (b) 0.00637

Factor the greatest common factor from the polynomial.

91. $3x^4-6x^3-18x^2$

Chapter 11 Graphs

Graph:

92. $y = 4x - 3$

93. $y = -3x$

94. $y = \frac{1}{2}x + 3$

95. $x - y = 6$

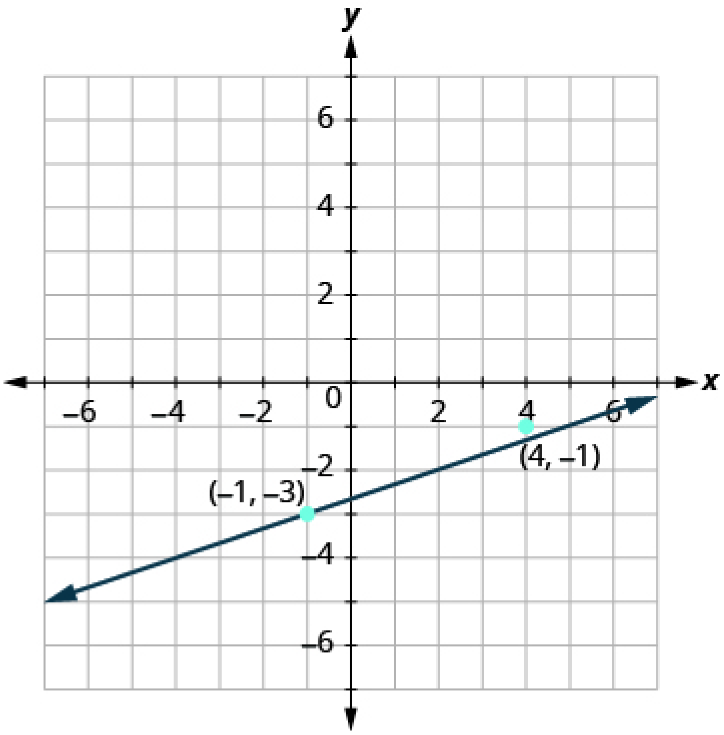
96. $y = -2$

97. Find the intercepts. $2x + 3y = 12$

Graph using the intercepts.

98. $2x - 4y = 8$

99. Find the slope of the line shown.



100. Use the slope formula to find the slope of the line between the points $(-5, -2), (3, 2)$.

101. Graph the line passing through the point $(-3, 4)$ and with slope $m = -\frac{1}{3}$.

Powers and Roots Tables

n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
1	1	1	1	1
2	4	1.414214	8	1.259921
3	9	1.732051	27	1.442250
4	16	2	64	1.587401
5	25	2.236068	125	1.709976
6	36	2.449490	216	1.817121
7	49	2.645751	343	1.912931
8	64	2.828427	512	2
9	81	3	729	2.080084
10	100	3.162278	1,000	2.154435
11	121	3.316625	1,331	2.223980
12	144	3.464102	1,728	2.289428
13	169	3.605551	2,197	2.351335
14	196	3.741657	2,744	2.410142

n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
15	225	3.872983	3,375	2.466212
16	256	4	4,096	2.519842
17	289	4.123106	4,913	2.571282
18	324	4.242641	5,832	2.620741
19	361	4.358899	6,859	2.668402
20	400	4.472136	8,000	2.714418
21	441	4.582576	9,261	2.758924
22	484	4.690416	10,648	2.802039
23	529	4.795832	12,167	2.843867
24	576	4.898979	13,824	2.884499
25	625	5	15,625	2.924018
26	676	5.099020	17,576	2.962496
27	729	5.196152	19,683	3
28	784	5.291503	21,952	3.036589
29	841	5.385165	24,389	3.072317
30	900	5.477226	27,000	3.107233
31	961	5.567764	29,791	3.141381

n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
32	1,024	5.656854	32,768	3.174802
33	1,089	5.744563	35,937	3.207534
34	1,156	5.830952	39,304	3.239612
35	1,225	5.916080	42,875	3.271066
36	1,296	6	46,656	3.301927
37	1,369	6.082763	50653	3.332222
38	1,444	6.164414	54,872	3.361975
39	1,521	6.244998	59,319	3.391211
40	1,600	6.324555	64,000	3.419952
41	1,681	6.403124	68,921	3.448217
42	1,764	6.480741	74,088	3.476027
43	1,849	6.557439	79,507	3.503398
44	1,936	6.633250	85,184	3.530348
45	2,025	6.708204	91,125	3.556893
46	2,116	6.782330	97,336	3.583048
47	2,209	6.855655	103,823	3.608826
48	2,304	6.928203	110,592	3.634241

n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
49	2,401	7	117,649	3.659306
50	2,500	7.071068	125,000	3.684031
51	2,601	7.141428	132,651	3.708430
52	2,704	7.211103	140,608	3.732511
53	2,809	7.280110	148,877	3.756286
54	2,916	7.348469	157,464	3.779763
55	3,025	7.416198	166,375	3.802952
56	3,136	7.483315	175,616	3.825862
57	3,249	7.549834	185,193	3.848501
58	3,364	7.615773	195,112	3.870877
59	3,481	7.681146	205,379	3.892996
60	3,600	7.745967	216,000	3.914868
61	3,721	7.810250	226,981	3.936497
62	3,844	7.874008	238,328	3.957892
63	3,969	7.937254	250,047	3.979057
64	4,096	8	262,144	4
65	4,225	8.062258	274,625	4.020726

n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
66	4,356	8.124038	287,496	4.041240
67	4,489	8.185353	300,763	4.061548
68	4,624	8.246211	314,432	4.081655
69	4,761	8.306624	328,509	4.101566
70	4,900	8.366600	343,000	4.121285
71	5,041	8.426150	357,911	4.140818
72	5,184	8.485281	373,248	4.160168
73	5,329	8.544004	389,017	4.179339
74	5,476	8.602325	405,224	4.198336
75	5,625	8.660254	421,875	4.217163
76	5,776	8.717798	438,976	4.235824
77	5,929	8.774964	456,533	4.254321
78	6,084	8.831761	474,552	4.272659
79	6,241	8.888194	493,039	4.290840
80	6,400	8.944272	512,000	4.308869
81	6,561	9	531,441	4.326749
82	6,724	9.055385	551,368	4.344481

n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
83	6,889	9.110434	571,787	4.362071
84	7,056	9.165151	592,704	4.379519
85	7,225	9.219544	614,125	4.396830
86	7,396	9.273618	636,056	4.414005
87	7,569	9.327379	658,503	4.431048
88	7,744	9.380832	681,472	4.447960
89	7,921	9.433981	704,969	4.464745
90	8,100	9.486833	729,000	4.481405
91	8,281	9.539392	753,571	4.497941
92	8,464	9.591663	778,688	4.514357
93	8,649	9.643651	804,357	4.530655
94	8,836	9.695360	830,584	4.546836
95	9,025	9.746794	857,375	4.562903
96	9,216	9.797959	884,736	4.578857
97	9,409	9.848858	912,673	4.594701
98	9,604	9.899495	941,192	4.610436
99	9,801	9.949874	970,299	4.626065

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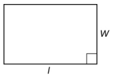
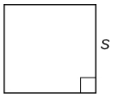
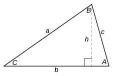
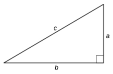
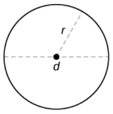
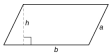
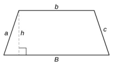
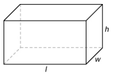
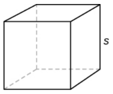
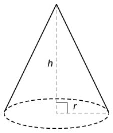
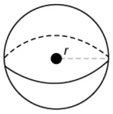
n	n^2	$\sqrt{n}$	n^3	$\sqrt[3]{n}$
100	10,000	10	1,000,000	4.641589

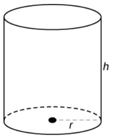
Geometric Formulas

Name	**Shape**	**Formulas**
Rectangle		Perimeter: $P = 2l + 2w$ Area: $A = lw$
Square		Perimeter: $P = 4s$ Area: $A = s^2$
Triangle		Perimeter: $P = a + b + c$ Area: $A = \frac{1}{2}bh$ Sum of Angles: $A + B + C = 180^\circ$
Right Triangle		Pythagorean Theorem: $a^2 + b^2 = c^2$ Area: $A = \frac{1}{2}ab$
Circle		Circumference: $C = 2\pi r$ or $C = \pi d$ Area: $A = \pi r^2$
Parellelogram		Perimeter: $P = 2a + 2b$ Area: $A = bh$

Name	**Shape**	**Formulas**
Trapezoid		Perimeter: $P = a + b + c + B$ Area: $A = \frac{1}{2}(B + b)h$

2 Dimensions

Name	**Shape**	**Formulas**
Rectangular Solid		Volume: $V = lwh$ Surface Area: $SA = 2lw + 2wh + 2hl$
Cube		Volume: $V = s^3$ Surface Area: $SA = 6s^2$
Cone		Volume: $V = \frac{1}{3}\pi r^2 h$ Surface Area: $SA = \pi r^2 + \pi r\sqrt{h^2 + r^2}$
Sphere		Volume: $V = \frac{4}{3}\pi r^3$ Surface Area: $SA = 4\pi r^2$

Name	**Shape**	**Formulas**
Right Circular Cylinder		Volume: $V = \pi r^2 h$ Surface Area: $SA = 2\pi r^2 + 2\pi rh$

3 Dimensions